

Flavor Changing Neutral Higgs Bosons in a Supersymmetric Extension based on a Q_6 Family Symmetry

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Abstract

A supersymmetric extension of the standard model based on the discrete Q_6 family symmetry is considered, and we investigate flavor-changing neutral current (FCNC) processes, especially those mediated by heavy flavor-changing neutral Higgs bosons. Because of the family symmetry the number of the independent Yukawa couplings is smaller than that of the observed quantities such as the Cabibbo-Kobayashi-Maskawa matrix and the quark masses, so that the FCNCs can be parametrized only by the mixing angles and masses of the Higgs fields. We focus our attention on the mass differences of the neutral K , D and B mesons. All the constraints including that from the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ can be satisfied, if the heavy Higgs bosons are heavier than ~ 1.5 TeV. If the constraint from ΔM_K is slightly relaxed, the heavy Higgs bosons can be as light as ~ 0.4 TeV, which is within the accessible range of LHC.

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I. INTRODUCTION

In recent studies on flavor symmetries ¹ it has become clear that a flavor symmetry can be realized at low energies. As long as this possibility is not excluded, theoretical as well as experimental searches for a low energy flavor symmetry should be continued. An important prediction of any viable low energy flavor symmetry, which is broken only spontaneously or at most softly, is the existence of multiple $SU(2)_L$ doublet Higgs fields, as one could read off from a sort of no-go theorem of [4]. This implies that there should exist several neutral Higgs fields that have flavor changing couplings to the fermions at the tree-level. Therefore, an observation of a non-standard flavor changing neutral current (FCNC) process, at LHC for instance, is not necessarily an indication of supersymmetry [5, 6].

In Ref. [7] a supersymmetric flavor model based on a dicyclic dihedral group Q_6 has been suggested. ² The main motivation there was to derive a modified Fritzsch mass matrix for the quarks from a flavor symmetry. With an assumption that CP is spontaneously broken, the model can fix six quark masses and four Cabibbo-Kobayashi-Maskawa (CKM) parameters in term of nine parameters of the model. It has been later realized in Refs. [21, 22] that through an appropriate change of the lepton assignment the leptonic sector can be brought into the same form as that of the model of [23, 24]. Then there are only seven parameters in the leptonic sector of the model to fix six lepton masses and six Maki-Nakagawa-Sakata (MNS) parameters. The discrete flavor group Q_6 is the smallest non-abelian group with which the above situation can be achieved.

However, it turned out that one has to introduce a certain set of $SU(2)_L \times U(1)_Y$ singlet fields and also additional abelian global symmetries to make the model viable. Nothing is wrong with this situation, but in this paper we would like to stress the minimal content of the Higgs fields and at the same time a “one + two” structure for each family; one Q_6 singlet and one Q_6 doublet for each family including the $SU(2)_L$ doublet Higgs fields. In Sec. II we will shed light upon the relation between the non-renormalization theorem and flavor symmetry, and will show that different flavor symmetries can be consistently introduced into a softly broken supersymmetric gauge theory. We will systematically investigate this possibility in a general framework. With this observation we will find in Sec. III that the one + two structure of family in a minimal Q_6 extension of the supersymmetric standard model (MSSM) can be consistently realized.

In Sec. IV we will consider the Higgs sector. Because of the one + two structure the Higgs sector is much simpler than that of [7, 21, 22], and therefore the sector can be investigated

¹ For recent reviews see, for instance, [1, 2, 3].

² Q_6 is one of Q_{2N} with $N = 2, 3, \dots$, which are the “covering groups” of the dihedral groups D_N [8, 9]. In recent years there are a number of interesting flavor models based on Q_{2N} and D_N . For instance, D_4 has been used as a flavor symmetry in Refs. [10, 11, 12, 13, 14], while D_5, D_6, D_7 and Q_4 have been considered in Refs. [15], [16], [17] and [18], respectively. See also Refs. [19, 20].

with much less assumptions. We will explicitly show that it is possible to fine tune the soft-supersymmetry-breaking (SSB) parameters so as to make the heavy Higgs bosons much heavier (several TeV) than M_Z and at the same time to obtain a desired size of spontaneous CP violation to reproduce the Kobayashi-Maskawa CP violating phase.

In Sec. V we will first calculate the unitary matrices that diagonalize the fermion mass matrices, which are needed to write down the Yukawa couplings in terms of mass eigenstates. We only briefly mention FCNCs and CP violations in the SSB sector and in the lepton sector, because detailed investigations on these subjects have been recently carried out in Ref. [22] and in Ref. [25], respectively. Instead we investigate FCNC processes mediated by neutral heavy Higgs fields. We concentrate on the constraints coming from the mass differences in the neutral meson systems, $\Delta M_K, \Delta M_{B_s}, \Delta M_{B_d}$ and ΔM_D , in a similar spirit as Refs. [26, 27, 28, 29, 30, 31, 32] and references therein. We express the relevant flavor changing-neutral Yukawa couplings in terms of the mass eigenstates, where except the phases the size of the Yukawa couplings are basically fixed. Allowed ranges in which the constraints are satisfied are shown in different figures. We find that the heavy Higgs bosons should be heavier than ~ 1.5 TeV, although it is possible to fine tune the parameters such that the constraints can be satisfied for lighter mass values.

Sect. VI is devoted for conclusion.

II. NON-RENORMALIZATION THEOREM AND FLAVOR SYMMETRY

A flavor symmetry can control the structure of the independent parameters of a theory. In supersymmetric theories, moreover, the non-renormalization theorem allows to suppress certain couplings and also to relate them with each other, without facing contradictions with renormalization. What is therefore the (technical) role of a flavor symmetry in supersymmetric theories? We recall that the D-terms are renormalized and the wave function renormalization can mix matter superfields Φ_i 's in general. Therefore, starting with diagonal kinetic terms $\Phi_i^* \Phi_i$ is not always consistent with renormalization. If a non-diagonal (infinite) kinetic term is induced, a corresponding non-diagonal counter term should be added. Then after the diagonalization the originally assumed structure of the couplings in the superpotential will receive large quantum corrections. In other words, we have in spite of the non-renormalization theorem more parameters in the superpotential, when written in terms of the bare fields, than originally assumed. The undesired mixing among Φ_i 's and large quantum corrections can be avoided if an appropriate flavor symmetry is present.

We will see below that the non-renormalization theorem and the renormalization properties of the soft-supersymmetry-breaking (SSB) terms allow us to introduce in a consistent manner different flavor symmetries for different sectors of a softly broken supersymmetric theory to control the independent parameters of the theory.

To be more specific, we consider an $N = 1$ supersymmetric gauge theory whose superpo-

tential is given by

$$W(\Phi) = W_Y(\Phi) + W_\mu(\Phi) , \quad (1)$$

with

$$W_Y(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k \text{ and } W_\mu(\Phi) = \frac{1}{2}\mu^{ij}\Phi_i\Phi_j . \quad (2)$$

The SSB Lagrangian can be written as

$$\begin{aligned} L(\Phi, W) = & - \left(\int d^2\theta \eta \left(\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}b^{ij}\Phi_i\Phi_j + \frac{1}{2}M_g W_A^\alpha W_{A\alpha} \right) + \text{h.c.} \right) \\ & - \int d^4\theta \tilde{\eta} \eta \overline{\Phi^j} (m^2)_j^i (e^{2gV})_i^k \Phi_k , \end{aligned} \quad (3)$$

where $\eta = \theta^2$, $\tilde{\eta} = \tilde{\theta}^2$ are the external spurion superfields and M_g is the gaugino mass. The β functions of the Y, μ, h and m^2 are given by Refs. [33]-[41]

$$\beta_Y^{ijk} = \gamma^i_l Y^{ljk} + \gamma^j_l Y^{ilk} + \gamma^k_l Y^{ijl} , \quad (4)$$

$$\beta_\mu^{ij} = \gamma^i_l \mu^{lj} + \gamma^j_l \mu^{il} , \quad (5)$$

$$\beta_h^{ijk} = \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} - 2\gamma_{1l}^i Y^{ljk} - 2\gamma_{1l}^j Y^{ilk} - 2\gamma_{1l}^k Y^{ijl} , \quad (6)$$

$$\beta_b^{ij} = \gamma^i_l b^{lj} + \gamma^j_l b^{il} - 2\gamma_{1l}^i \mu^{lj} - 2\gamma_{1l}^j \mu^{il} , \quad (7)$$

$$(\beta_{m^2})_j^i = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma_j^i , \quad (8)$$

$$\mathcal{O} = \left(M_g g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right) , \quad (9)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M_g|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn} \frac{\partial}{\partial Y^{lmn}} , \quad (10)$$

where $(\gamma_1)_j^i = \mathcal{O}\gamma_j^i$, $Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)_l^i Y^{ljk} + (m^2)_l^j Y^{ilk} + (m^2)_l^k Y^{ijl} , \quad (11)$$

$$X = \frac{-|M_g|^2 C(G) + \sum_l m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} . \quad (12)$$

Here X of (12) is the expression in the renormalization scheme of Novikov *et al.* [42], $T(R_l)$ is the Dynkin index of R_l , and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . From Eqs. (4)-(12) we now derive the hierarchical structure of the renormalization properties of the theory, which is basically the Symanzik theorem applied to softly broken supersymmetric gauge theories:

1. The (infinite) renormalization of the supersymmetric parameters Y^{ijk} , μ^{ij} is not influenced by the SSB terms, in accord with the definition of the SSB terms.
2. The (infinite) renormalization of the tri-linear couplings h^{ijk} does not depend on μ^{ij} . It is also independent on $(m^2)_j^i$ and b^{ij} .

3. The (infinite) renormalization of the soft scalar masses $(m^2)_j^i$ does not depend on b^{ij} and μ^{ij} , as one can see from Eqs. (8)- (12).
4. The (infinite) renormalization of b^{ij} does not depend $(m^2)_j^i$ and h^{ijk} , which is the consequence of (7).

Because of these renormalization properties we can consistently introduce different symmetries for different sectors.

To begin with we assume the existence of a flavor symmetry in the Yukawa sector which protects the mixing (of the wave function renormalization) among the matter superfields Φ_i 's. ³ This implies that the anomalous dimensions γ_j^i are diagonal, i.e.,

$$\gamma_j^i = \delta_j^i \gamma_j . \quad (13)$$

Then Eqs. (4) -(8) become

$$\beta_Y^{ijk} = Y^{ijk}(\gamma_i + \gamma_j + \gamma_k) , \quad \beta_\mu^{ij} = \mu^{ij}(\gamma_i + \gamma_j) , \quad (14)$$

$$\beta_h^{ijk} = (h^{ijk} - 2Y^{ijk}\mathcal{O})(\gamma_i + \gamma_j + \gamma_k) , \quad \beta_b^{ij} = (b^{ij} - 2\mu^{ij}\mathcal{O})(\gamma_i + \gamma_j) , \quad (15)$$

$$(\beta_{m^2})_l = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma_l , \quad (16)$$

with $\tilde{Y}^{ijk} = Y^{ijk}(m_i^2 + m_j^2 + m_k^2)$. From these equations we observe:

- (a) The μ sector can have a flavor symmetry which is different from the flavor symmetry of the Yukawa sector if both symmetries are compatible with respect to renormalization of μ_{ij} .
- (b) It is consistent to introduce into the tri-linear couplings the same flavor symmetry as that of the Yukawa couplings, even if it is violated in other sectors.
- (c) The flavor symmetry which protects the mixing among Φ_i 's ensures that $(m^2)_j^i$ is diagonal. If the Yukawa couplings and tri-linear couplings have the flavor symmetry, the soft scalar mass terms, too, can have the flavor symmetry, even if the μ and b terms do not respect the flavor symmetry.
- (d) The b terms associated with the μ terms should always exist (see (16)). But the b sector has no influence on the infinite renormalization of the parameters in other sectors. So the violation of a symmetry in the b sector is absolutely soft.

In the next section we reconsider the supersymmetric flavor model of [7, 21, 22] along the line of thought about a flavor symmetry in this section.

³ We also assume that the flavor symmetry is not gauged.

| | Q | Q_3 | U^c, D^c | U_3^c, D_3^c | L | L_3 | E^c, N^c | E_3^c | N_3^c | H^u, H^d | H_3^u, H_3^d |
|-------|----------------|--------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|--------------------|----------------|--------------------|
| Q_6 | $\mathbf{2}_1$ | $\mathbf{1}_{+,2}$ | $\mathbf{2}_2$ | $\mathbf{1}_{-,1}$ | $\mathbf{2}_2$ | $\mathbf{1}_{+,0}$ | $\mathbf{2}_2$ | $\mathbf{1}_{+,0}$ | $\mathbf{1}_{-,3}$ | $\mathbf{2}_2$ | $\mathbf{1}_{-,1}$ |
| R | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $+$ | $+$ |

TABLE I: The $Q_6 \times R$ assignment of the chiral matter supermultiplets, where R is the R parity. The group theory notation is given in Ref. [7].

III. THE MODEL

The supersymmetric flavor model of [7, 21, 22] is based on a dicyclic dihedral group Q_6 . If CP is spontaneously broken, the nine parameters of the model express six quark masses and four CKM parameters. In the leptonic sector there are only seven parameters to fix six lepton masses and six MNS parameters. As we announced in the introduction we would like to stress the one + two structure for each family; a Q_6 singlet and a Q_6 doublet for each family including the $SU(2)_L$ doublet Higgs fields.

A. The Yukawa sector

As in the original model of [7, 21, 22] we assume that the flavor symmetry of the Yukawa sector is based on Q_6 . In Table I we write the Q_6 assignment of the quark, lepton and Higgs chiral supermultiplets,⁴ where Q, Q_3, L, L_3 and H^u, H_3^u, H^d, H_3^d stand for $SU(2)_L$ doublet supermultiplets for the quarks, leptons and Higgs bosons, respectively. Similarly, $SU(2)_L$ singlet supermultiplets for quarks, charged leptons and neutrinos are denoted by $U^c, U_3^c, D^c, D_3^c, E^c, E_3^c$ and N^c, N_3^c . From Table I we see that the one+two structure of family is realized, and because of this structure the Q_6 flavor symmetry can ensure that no non-diagonal kinetic term can be induced. So (13) is satisfied.

We then write down the most general, renormalizable, $Q_6 \times R$ invariant superpotential W (R is the R parity.):

$$W_Y = W_Q + W_L , \quad (17)$$

where

$$W_Q = \sum_{I,i,j,k=1,2,3} (Y_{ij}^{uI} Q_i U_j^c H_I^u + Y_{ij}^{dI} Q_i D_j^c H_I^d) , \quad (18)$$

$$W_L = \sum_{I,i,j,k=1,2,3} (Y_{ij}^{eI} L_i E_j^c H_I^u + Y_{ij}^{\nu I} L_i N_j^c H_I^u) . \quad (19)$$

⁴ The same model exists for Q_{2N} if N is odd and a multiple of 3.

The Yukawa matrices Y 's are given by

$$\begin{aligned}\mathbf{Y}^{u1(d1)} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_b^{u(d)} \\ 0 & Y_{b'}^{u(d)} & 0 \end{pmatrix}, \quad \mathbf{Y}^{u2(d2)} = \begin{pmatrix} 0 & 0 & Y_b^{u(d)} \\ 0 & 0 & 0 \\ -Y_{b'}^{u(d)} & 0 & 0 \end{pmatrix}, \\ \mathbf{Y}^{u3(d3)} &= \begin{pmatrix} 0 & Y_c^{u(d)} & 0 \\ Y_c^{u(d)} & 0 & 0 \\ 0 & 0 & Y_a^{u(d)} \end{pmatrix},\end{aligned}\tag{20}$$

$$\mathbf{Y}^{e1} = \begin{pmatrix} -Y_c^e & 0 & Y_b^e \\ 0 & Y_c^e & 0 \\ Y_{b'}^e & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{e2} = \begin{pmatrix} 0 & Y_c^e & 0 \\ Y_c^e & 0 & Y_b^e \\ 0 & Y_{b'}^e & 0 \end{pmatrix}, \quad \mathbf{Y}^{e3} = 0,\tag{21}$$

$$\begin{aligned}\mathbf{Y}^{\nu1} &= \begin{pmatrix} -Y_c^\nu & 0 & 0 \\ 0 & Y_c^\nu & 0 \\ Y_{b'}^\nu & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{\nu2} = \begin{pmatrix} 0 & Y_c^\nu & 0 \\ Y_c^\nu & 0 & 0 \\ 0 & Y_{b'}^\nu & 0 \end{pmatrix}, \\ \mathbf{Y}^{\nu3} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_a^\nu \end{pmatrix}.\end{aligned}\tag{22}$$

All the parameters appearing above are real, because we assume that CP is spontaneously broken. We will shortly come back to this issue.

B. The μ sector

The most general $Q_6 \times R$ invariant renormalizable μ part of the superpotential is

$$W_\mu^{(Q_6)} = \mu H_I^u H_I^d + \frac{m}{2} N_I^c N_I^c.\tag{23}$$

Note that no mass terms for $H_3^{u,d}$ and N_3^c are allowed by Q_6 and that the superpotential $W_\mu^{(Q_6)}$ has an accidental $O(2)$ symmetry. For phenomenological reasons we however need mass terms for $H_3^{u,d}$ and N_3^c . Therefore, we assume that the flavor symmetry of the μ sector is $O(2)$ and that $H_3^{u,d}$ and N_3^c are singlets of $O(2)$, and add

$$W_\mu^{(Q_6)} = \mu_3 H_3^u H_3^d + \frac{m_3}{2} N_3^c N_3^c\tag{24}$$

to (23). Then the total μ part of the superpotential is $W_\mu = W_\mu^{(Q_6)} + W_\mu^{(Q_6)}$. The $O(2) \times R$ symmetry of W_μ is compatible with $Q_6 \times R$ of the Yukawa sector, because Q_6 ensures

$$\gamma_{H_1^u} = \gamma_{H_2^u} \text{ and } \gamma_{H_1^d} = \gamma_{H_2^d}.\tag{25}$$

C. Soft-supersymmetry-breaking sector

1. The tri-linear couplings and soft scalar mass terms

We require that the tri-linear couplings and soft scalar mass terms have the same flavor symmetry as that of the Yukawa sector, that is, $Q_6 \times R$. Therefore, the tri-linear couplings and soft scalar mass matrices have the following form:

$$\mathbf{h}_{ij}^k = A_{ij} \mathbf{Y}_{ij}^k, \quad k = u1, u2, \dots, \nu_3, \quad (26)$$

where \mathbf{Y}_{ij}^k are given in (20)-(22), and

$$\mathbf{m}^2 \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & f \end{pmatrix}. \quad (27)$$

for all the bosonic scalar partners. This is very crucial to suppress FCNCs in the SSB sector as we will see later on.

2. The b terms

The b sector should contain at least terms which correspond to the μ terms $W_\mu = W_\mu^{(Q_6)} + W_\mu^{(Q_6)}$, where $W_\mu^{(Q_6)}$ and $W_\mu^{(Q_6)}$ are given in (23) and (24), respectively, i.e.

$$\mathcal{L}_b^{(O_2)} = b \hat{H}_I^u \hat{H}_I^d + b_{33} \hat{H}_3^u \hat{H}_3^d + b_N \hat{N}_I^c \hat{N}_I^c + b_{N_3} \hat{N}_3^c \hat{N}_3^c + h.c. \quad (28)$$

(The hatted fields are bosonic components.) Because of the $O(2)$ symmetry in the μ and b sectors and the Q_6 symmetry in the soft scalar mass terms, the Higgs scalar potential also respects the $O(2)$ symmetry, so that there is a Nambu-Goldstone boson corresponding to this symmetry because the $O(2)$ symmetry the gauge symmetry is spontaneously broken, together with $SU(2)_L \times U(1)_Y$. Moreover, we face the domain wall problem when the discrete flavor symmetries are spontaneously broken. To overcome these problems we add terms which explicitly break O_2 down to Z_2 :

$$\begin{aligned} \mathcal{L}_b^{(O_2)} = & b_{++} \hat{H}_+^u \hat{H}_+^d + b_{--} \hat{H}_-^u \hat{H}_-^d + b_{+3} \hat{H}_+^u \hat{H}_3^d + b_{3+} \hat{H}_3^u \hat{H}_+^d \\ & + b_{N_+} \hat{N}_+^c \hat{N}_+^c + b_{N_-} \hat{N}_-^c \hat{N}_-^c + \hat{N}_3^c \hat{N}_3^c + h.c., \end{aligned} \quad (29)$$

where

$$H_\pm^{u,d} = \frac{1}{\sqrt{2}}(H_1^{u,d} \pm H_2^{u,d}), \quad N_\pm^c = \frac{1}{\sqrt{2}}(N_1^c \pm N_2^c). \quad (30)$$

($H_+^{u,d}, H_3^{u,d}, N_+^c$ and N_3^c are Z_2 even, while $H_-^{u,d}$ and N_-^c are Z_2 odd.) This Z_2 is indeed broken by the Yukawa and tri-linear couplings, but is compatible with Q_6 , i.e., $\gamma_{H_1^{u,d}} = \gamma_{H_2^{u,d}}$.

| | Y, h | m | μ sector | b terms |
|-------|-------------|------------|--------------|------------|
| Q_6 | \bigcirc | \bigcirc | \times | \times |
| O_2 | \times | \bigcirc | \bigcirc | \times |
| Z_2 | \times | \bigcirc | \bigcirc | \bigcirc |
| CP | \bigcirc | \bigcirc | \bigcirc | \times |
| R | \bigcirc | \bigcirc | \bigcirc | \bigcirc |

TABLE II: The symmetry of the different sectors. **Y, h** and **m** stand for the Yukawa, tri-linear and soft scalar mass sector, respectively. Q_6 ensures that all the anomalous dimensions γ 's are diagonal, and that the two components of a Q_6 doublet have a same anomalous dimension. Therefore, Q_6 in the Yukawa and tri-linear sectors and O_2 in the μ sector are compatible with each other. O_2 in the soft scalar mass sector is accidental. Z_2 is a subgroup of O_2 , which implies the compatibility of O_2 and Z_2 . CP is explicitly broken only by the b terms, which is (super) soft because the propagation of its violation to the other sectors is calculable and small. So, all the symmetries are compatible with each other.

We allow the b parameters to be complex, because CP can not be broken if all the b parameters are real as we will find in the next subsection. So CP is explicitly, but only softly broken in this sector. In Table II we give the symmetry of the each sector.

IV. THE HIGGS SECTOR

A. The Higgs potential

Given the $O(2) \times R$ invariant superpotential W_μ in the μ sector (23) and (24) along with the $Q_6 \times R$ invariant soft scalar masses (27) and the $Z_2 \times R$ invariant b terms (28) and (29), we can now write down the scalar potential. For simplicity we assume that only the neutral scalar components (denoted by a superscript 0) of the Higgs supermultiplets acquire VEVs:

$$\begin{aligned}
V = & m_{H_+^u}^2 (|\hat{H}_+^{0u}|^2 + |\hat{H}_-^{0u}|^2) + m_{H_+^d}^2 (|\hat{H}_+^{0d}|^2 + |\hat{H}_-^{0d}|^2) + m_{H_3^u}^2 |\hat{H}_3^{0u}|^2 + m_{H_3^d}^2 |\hat{H}_3^{0d}|^2 \\
& + \frac{1}{8}(\frac{3}{5}g_1^2 + g_2^2)(|\hat{H}_+^{0u}|^2 + |\hat{H}_-^{0u}|^2 + |\hat{H}_3^{0u}|^2 - |\hat{H}_+^{0d}|^2 - |\hat{H}_-^{0d}|^2 - |\hat{H}_3^{0d}|^2)^2 \\
& + \left[b'_{++} \hat{H}_+^{0u} \hat{H}_+^{0d} + b'_{--} \hat{H}_-^{0u} \hat{H}_-^{0d} + b_{+3} \hat{H}_+^{0u} \hat{H}_3^{0d} + b_{3+} \hat{H}_3^{0u} \hat{H}_+^{0d} + b_{33} \hat{H}_3^{0u} \hat{H}_3^{0d} + h.c. \right], \quad (31)
\end{aligned}$$

where $b'_{++(-)} = b + b_{++(-)}$, $g_{1,2}$ are the gauge coupling constants for the $U(1)_Y$ and $SU(2)_L$ gauge groups, and H_\pm 's are defined in (30). Note that the scalar potential (31) has the same Z_2 symmetry as that of the b sector. (H_+ 's and H_3 's are Z_2 even, and H_- 's are Z_2 odd.) Therefore,

$$\langle \hat{H}_-^{0u,d} \rangle = 0, \langle \hat{H}_+^{0u,d} \rangle = \frac{v_+^{u,d}}{\sqrt{2}} \exp i\theta_+^{u,d}, \langle \hat{H}_3^{0u,d} \rangle = \frac{v_3^{u,d}}{\sqrt{2}} \exp i\theta_3^{u,d} \quad (32)$$

can become a local minimum, where we assume that $v_+^{u,d}$ and $v_3^{u,d}$ are real. We recall that the Z_2 is an accidental symmetry expect for the b sector.⁵ Therefore, the VEV structure (32) is stable against (infinite) renormalization.

We investigate whether the potential energy at the VEV (32) can become negative so that $SU(2)_L \times U(1)_Y$ is spontaneously broken. To this end we consider the quadratic part of the scalar potential

$$V^{(2)} = \mathcal{H}^I \mathcal{M}_{IJ} \mathcal{H}^J, \quad (33)$$

where

$$\mathcal{M} = \begin{pmatrix} m_{H_+^u}^2 & 0 & \Re(b'_{++}) & -\Im(b'_{++}) & 0 & 0 & \Re(b_{+3}) & -\Im(b_{+3}) \\ 0 & m_{H_+^u}^2 & -\Im(b'_{++}) & -\Re(b'_{++}) & 0 & 0 & -\Im(b_{+3}) & -\Re(b_{+3}) \\ \Re(b'_{++}) & -\Im(b'_{++}) & m_{H_+^d}^2 & 0 & \Re(b_{3+}) & -\Im(b_{3+}) & 0 & 0 \\ -\Im(b'_{++}) & -\Re(b'_{++}) & 0 & m_{H_+^d}^2 & -\Im(b_{3+}) & -\Re(b_{3+}) & 0 & 0 \\ 0 & 0 & \Re(b_{3+}) & -\Im(b_{3+}) & m_{H_3^u}^2 & 0 & \Re(b_{33}) & -\Im(b_{33}) \\ 0 & 0 & -\Im(b_{3+}) & -\Re(b_{3+}) & 0 & m_{H_3^u}^2 & -\Im(b_{33}) & -\Re(b_{33}) \\ \Re(b_{+3}) & -\Im(b_{+3}) & 0 & 0 & \Re(b_{33}) & -\Im(b_{33}) & m_{H_3^d}^2 & 0 \\ -\Im(b_{+3}) & -\Re(b_{+3}) & 0 & 0 & -\Im(b_{33}) & -\Re(b_{33}) & 0 & m_{H_3^d}^2 \end{pmatrix}, \quad (34)$$

and

$$\mathcal{H} = (\Re(\hat{H}_+^{0u}), \Im(\hat{H}_+^{0u}), \Re(\hat{H}_+^{0d}), \Im(\hat{H}_+^{0d}), \Re(\hat{H}_3^{0u}), \Im(\hat{H}_3^{0u}), \Re(\hat{H}_3^{0d}), \Im(\hat{H}_3^{0d})). \quad (35)$$

We find that all the eigenvalues of \mathcal{M} are doubly generate, and that two orthogonal eigenvectors of the same eigenvalue can be always written in the form

$$\vec{u}_A = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) \text{ and } \vec{u}_B = (u_2, -u_1, -u_4, u_3, u_6, -u_5, -u_8, u_7). \quad (36)$$

This is due to the $U(1)_Y$ gauge invariance: All the directions defined by a linear combination of \vec{u}_A and \vec{u}_B are physically equivalent. If all the imaginary parts of b 's vanish, then we find $u_2 = u_4 = u_6 = u_8 = 0$, which means that CP can not be spontaneously broken, because the imaginary parts $\Im(\mathcal{H}_I)$ along the direction defined by $(u_1, 0, u_3, 0, u_5, 0, u_7, 0)$ stay at zero. So at least one of b 's should be complex so that CP is spontaneously broken.⁶ The product of the four independent eigenvalues is $\det \mathcal{M}$. Therefore, if $\det \mathcal{M}$ is negative, one

⁵ It is accidental in the part of (31) coming from the D-terms (the second line). The Q_6 invariant soft scalar mass terms respect automatically this Z_2 , although it is not contained in Q_6 . This Z_2 is a part of the $O(2)$ symmetry of the μ sector, which is only softly broken down to the Z_2 in the b sector.

⁶ Spontaneous CP violation in supersymmetric models and two Higgs doublet models have been discussed in Refs. [43]-[49], Ref. [50] and references therein.

or three independent eigenvalues are negative. If $\det \mathcal{M}$ is positive, there may be zero, two or four negative eigenvalues. In this case one should compute the eigenvalues explicitly. A local minimum lies along the direction of a negative eigenvalue. Further, the potential (31) along the D-term flat direction should not be unbounded below. This condition requires

$$\begin{aligned} m_{H_+^u}^2 + m_{H_+^d}^2 - 2|b'_{++}| &> 0, \quad m_{H_+^u}^2 + m_{H_+^d}^2 - 2|b'_{--}| > 0, \quad m_{H_+^u}^2 + m_{H_3^d}^2 - 2|b_{+3}| > 0, \\ m_{H_3^u}^2 + m_{H_+^d}^2 - 2|b_{3+}| &> 0, \quad m_{H_3^u}^2 + m_{H_3^d}^2 - 2|b_{33}| > 0. \end{aligned} \quad (37)$$

We have to make the flavor changing neutral Higgs bosons sufficiently heavy to suppress FCNCs. (This will be discussed in Sec. V.) So we need a certain fine tuning among the SSB parameters, because the size of the VEVs is bounded from above. To achieve this situation, we have to so fine tune the parameters that one negative eigenvalue at the origin of the potential becomes very small.⁷ Then the potential energy falls only slowly when moving from the origin, and the quartic terms in the potential (31) coming from the D-terms start to dominate, so that the energy scale of the VEVs at the bottom of the potential can be much smaller than the energy scale of the SSB parameters. Here is such an example:

$$\begin{aligned} \Im(b_{++})/\Re(b'_{++}) &= 0.747, \quad \Re(b_{33})/\Re(b'_{++}) = 0.852, \quad \Im(b_{33})/\Re(b'_{++}) = 1.399, \\ \Re(b_{+3})/\Re(b'_{++}) &= 0.667, \quad \Im(b_{+3})/\Re(b'_{++}) = 0.31, \\ \Re(b_{3+})/\Re(b'_{++}) &= 1.3, \quad \Im(b_{3+})/\Re(b'_{++}) = 0.42, \\ m_{H_+^u}^2/\Re(b'_{++}) &= 3.13, \quad m_{H_+^d}^2/\Re(b'_{++}) = 2.69, \\ m_{H_3^u}^2/\Re(b'_{++}) &= 1.39, \quad m_{H_3^d}^2/\Re(b'_{++}) = 5.93. \end{aligned} \quad (38)$$

The four independent eigenvalues are -5.4×10^{-5} , 2.27, 4.16, 6.70 in the unit of b'_{++} , and two eigenvectors for the smallest eigenvalue correspond to

$$\begin{aligned} u_1 &= -0.1070, u_2 = 0.2232, u_3 = 0.4091, u_4 = 0.3081, \\ u_5 &= -0.4216, u_6 = 0.6636, u_7 = 0.2408, u_8 = 0.0154, \end{aligned} \quad (39)$$

where u 's are defined in (36). Along the direction defined by (39) the potential energy falls very slowly when moving from the origin. So the $SU(2)_L \times U(1)_Y$ invariant point is a saddle point, and we find that the size of $\sqrt{b'_{++}}$ may be estimated as

$$\sqrt{b'_{++}} \simeq \left(\frac{0.13(g_2^2 + 3g_1^2/5)/8}{5.4 \times 10^{-5}} \right)^{1/2} \times (246 \text{ GeV}) \simeq 3.2 \text{ TeV}. \quad (40)$$

CP is also spontaneously broken, because it is not possible to obtain a vector of the form $(\bullet, 0, \bullet, 0, \bullet, 0, \bullet, 0)$ through a linear combination of \vec{u}_A and \vec{u}_B for (39). Therefore, the angle θ_q that enters in the calculation of the CKM (given in (67)) is non-zero for (39). We find:

$$\begin{aligned} \theta_q &= \theta_+^u - \theta_+^d - \theta_3^u + \theta_3^d \\ &= \arctan(u_2/u_1) - \arctan(u_4/u_3) - \arctan(u_6/u_5) + \arctan(u_8/u_7) \simeq -0.701, \end{aligned} \quad (41)$$

⁷ By one eigenvalue we mean one of four eigenvalues. All the eigenvalues are doubly degenerate.

which is the size of θ_q we need to produce the correct CKM parameters as we will see in Sec. V.

B. The heavy neutral Higgs fields

Now redefine the Higgs fields as follows: First we define the tilde fields

$$\tilde{H}_+^{0u,0d} = \hat{H}_+^{0u,0d} \exp -i\theta_+^{u,d}, \quad \tilde{H}_3^{0u,0d} = \hat{H}_3^{0u,0d} \exp -i\theta_3^{u,d}, \quad (42)$$

and then

$$\phi_L^u = \cos \gamma^u \tilde{H}_3^{0u} + \sin \gamma^u \tilde{H}_+^{0u}, \quad \phi_H^u = -\sin \gamma^u \tilde{H}_3^{0u} + \cos \gamma^u \tilde{H}_+^{0u}, \quad (43)$$

where

$$\cos \gamma^u = \frac{v_3^u}{\sqrt{(v_3^u)^2 + (v_+^u)^2}}, \quad \sin \gamma^u = \frac{v_+^u}{\sqrt{(v_3^u)^2 + (v_+^u)^2}}, \quad (44)$$

and similarly for the down sector. As we see from (44), only ϕ_L^u and ϕ_L^d have a nonvanishing VEV, which we denote by

$$\langle \phi_L^{u,d} \rangle = \frac{\sqrt{(v_3^{u,d})^2 + (v_+^{u,d})^2}}{\sqrt{2}} = \frac{v_{u,d}}{\sqrt{2}}. \quad (45)$$

The neutral light and heavy scalars of the MSSM are given by

$$\frac{1}{\sqrt{2}}(v + h) = \text{Re}(\phi_L^{d*}) \cos \beta + \text{Re}(\phi_L^u) \sin \beta, \quad (46)$$

$$\frac{1}{\sqrt{2}}(H + iA) = -(\phi_L^{d*}) \sin \beta + (\phi_L^u) \cos \beta, \quad (47)$$

where as in the MSSM

$$v = \sqrt{v_u^2 + v_d^2}, \quad \tan \beta = \frac{v_u}{v_d}. \quad (48)$$

As in the case of the MSSM, the couplings of $\phi_L^{u,d}$ are flavor-diagonal, and so we do not have to consider them below when discussing FCNCs. Therefore, only the heavy fields $\hat{H}_-^{0u,0d} = \phi_-^{u,d}$ and $\phi_H^{u,d}$ can have flavor-changing couplings. Their mass matrix can be written as

$$\begin{pmatrix} m_{\phi_I^u}^2 & 0 & 0 & b_I^* \\ 0 & m_{\phi_I^u}^2 & b_I & 0 \\ 0 & b_I^* & m_{\phi_I^d}^2 & 0 \\ b_I & 0 & 0 & m_{\phi_I^d}^2 \end{pmatrix} \quad (49)$$

in the $(\phi_I^u, \phi_I^{u*}, \phi_I^d, \phi_I^{d*})$ basis, where $I = -, H$,

$$\begin{aligned} m_{\phi_{-}^{u,d}}^2 &= m_{H_{-}^{u,d}}^2, b_{-} = b'_{--}, m_{\phi_H^{u,d}}^2 = m_{H_{+}^{u,d}}^2 \cos^2 \gamma^{u,d} + m_{H_3^{u,d}}^2 \sin^2 \gamma^{u,d}, \\ b_H &= b'_{++} e^{-i(\theta_+^u + \theta_+^d)} \cos \gamma^u \cos \gamma^d - b_{+3} \cos \gamma^u \sin \gamma^d e^{-i(\theta_+^u + \theta_3^d)} \\ &\quad - b_{3+} \sin \gamma^u \cos \gamma^d e^{-i(\theta_3^u + \theta_+^d)} + b_{33} \sin \gamma^u \sin \gamma^d e^{-i(\theta_3^u + \theta_3^d)}, \end{aligned} \quad (50)$$

and the mass parameters on the rhs are given in (31) and $\gamma^{u,d}$ are defined in (44). The inverse of the matrix (49) is given by

$$\frac{1}{(M_{I1})^2 (M_{I2})^2} \begin{pmatrix} m_{\phi_I^d}^2 & 0 & 0 & -b_I^* \\ 0 & m_{\phi_I^d}^2 & -b_I & 0 \\ 0 & -b_I^* & m_{\phi_I^u}^2 & 0 \\ -b_I & 0 & 0 & m_{\phi_I^u}^2 \end{pmatrix}, \quad (I = -, H), \quad (51)$$

where $M_{1,2}$ are approximate pole masses and given by

$$(M_{I1(2)})^2 = \frac{1}{2}(m_{\phi_I^u}^2 + m_{\phi_I^d}^2) \left(1 + (-) \left[\frac{4|b_I|^2 + (m_{\phi_I^u}^2 - m_{\phi_I^d}^2)^2}{(m_{\phi_I^u}^2 + m_{\phi_I^d}^2)^2} \right]^{1/2} \right), \quad (52)$$

and we find

$$(M_{I1})^2 (M_{I2})^2 = -|b_I|^2 + m_{\phi_I^u}^2 m_{\phi_I^d}^2. \quad (53)$$

(51) is the inverse propagator at the zero momentum. We will be using it later on. For the parameter values in the example (39) we find

$$\begin{aligned} \tan \gamma^u &= 0.315, \quad \tan \gamma^d = 2.122, \quad \tan \beta = -1.456, \\ M_{H1} &= 2.31 \sqrt{b'_{++}} \simeq 7.3 \text{ TeV}, \quad M_{H2} = 1.72 \sqrt{b'_{++}} \simeq 5.5 \text{ TeV}, \end{aligned} \quad (54)$$

where we have used (40). So, what we have numerically shown in A and B in this section is that it is possible to fine tune the SSB parameters so as to make the heavy Higgs bosons much heavier than M_Z (see (54)) and at the same time to obtain a desired size of spontaneous CP violation (see (41)).

V. FCNCS

A. The physical quarks and leptons

From the Yukawa interactions (18) and (19) along with the form of the VEVs (32) we obtain the fermion mass matrices.

1. Quark sector

The quark mass matrices are given by

$$\mathbf{m}^u = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & Y_b^u v_+^u e^{-i\theta_+^u} \\ \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & 0 & Y_b^u v_+^u e^{-i\theta_+^u} \\ -Y_{b'}^u v_+^u e^{-i\theta_+^u} & Y_{b'}^u v_+^u e^{-i\theta_+^u} & \sqrt{2}Y_a^u v_3^u e^{-i\theta_3^u} \end{pmatrix}, \quad (55)$$

$$\mathbf{m}^d = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & Y_b^d v_+^d e^{-i\theta_+^d} \\ \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & 0 & Y_b^d v_+^d e^{-i\theta_+^d} \\ -Y_{b'}^d v_+^d e^{-i\theta_+^d} & Y_{b'}^d v_+^d e^{-i\theta_+^d} & \sqrt{2}Y_a^d v_3^d e^{-i\theta_3^d} \end{pmatrix}. \quad (56)$$

Then using the phase matrices defined below

$$R_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad R_R = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad (57)$$

$$P_L^u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta^u) & 0 \\ 0 & 0 & \exp(i\Delta\theta^u) \end{pmatrix}, \quad (58)$$

$$P_R^u = \begin{pmatrix} \exp(i2\Delta\theta^u) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(i\Delta\theta^u) \end{pmatrix} \exp(i\theta_3^u), \quad (59)$$

$$\Delta\theta^u = \theta_3^u - \theta_+^u, \quad (60)$$

and similarly for the down sector, we can bring \mathbf{m}^u into a real form

$$\hat{\mathbf{m}}^u = P_L^{u\dagger} R_L^T \mathbf{m}^u R_R P_R^u = m_t \begin{pmatrix} 0 & q_u/y_u & 0 \\ -q_u/y_u & 0 & b_u \\ 0 & b'_u & y_u^2 \end{pmatrix}. \quad (61)$$

The mass matrix $\hat{\mathbf{m}}^u$ can then be diagonalized as

$$O_L^{uT} \hat{\mathbf{m}}^u O_R^u = \begin{pmatrix} m_u & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad (62)$$

and similarly for \mathbf{m}^d , where $O_{L,R}^{u,d}$ are orthogonal matrices. So the mass eigenstates $u'_{iL} = (u'_L, c'_L, t'_L)$ etc. can be obtained from

$$u_L = U_L^u u'_{iL}, u_R = U_R^u u'_{iR}, d_L = U_L^d d'_{iL}, d_R = U_R^d d'_{iR}, \quad (63)$$

where

$$U_{uL(R)} = R_{L(R)} P_{L(R)}^u O_{L(R)}^u. \quad (64)$$

Therefore, the CKM matrix V_{CKM} is given by

$$V_{\text{CKM}} = O_L^{uT} R_L^T R_R P_L^{u\dagger} O_L^d = O_L^{uT} P_q O_L^d \quad (65)$$

where

$$P_q = P_L^{u\dagger} P_L^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta_q) & 0 \\ 0 & 0 & \exp(i\Delta\theta_q) \end{pmatrix}. \quad (66)$$

For the set of the parameters

$$\begin{aligned} \theta_q &= \theta_3^d - \theta_+^d - \theta_3^u + \theta_+^u = -0.7, q_u = 0.0001799, b_u = 0.05979, b'_u = 0.07054, \\ y_u &= 0.99786, q_d = 0.003784, b_d = 0.03268, b'_d = 0.4620, y_d = -0.9415, \end{aligned} \quad (67)$$

we obtain

$$m_u/m_t = 0.766 \times 10^{-5}, m_c/m_t = 4.23 \times 10^{-3}, m_d/m_b = 0.895 \times 10^{-3}, m_s/m_b = 1.60 \times 10^{-2},$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.9740 & 0.2266 & 0.00362 \\ 0.2265 & 0.9731 & 0.0417 \\ 0.00849 & 0.0410 & 0.9991 \end{pmatrix}, \quad |V_{td}/V_{ts}| = 0.207, \quad (68)$$

$$\sin 2\beta(\phi_1) = 0.690, \quad \gamma(\phi_3) = 63.4^\circ. \quad (69)$$

The experimental values to be compared are [51] (see also [52]):

$$|V_{\text{CKM}}^{\text{exp}}| = \begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & 0.00396^{+0.00009}_{-0.00009} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & 0.04221^{+0.00010}_{-0.00080} \\ 0.00814^{+0.00032}_{-0.00064} & 0.04161^{+0.00012}_{-0.00078} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix},$$

$$\sin 2\beta(\phi_1) = 0.687 \pm 0.032, \quad |V_{td}/V_{ts}| = 0.208^{+0.008}_{-0.006}. \quad (70)$$

The quark masses at M_Z are given by [53]

$$\begin{aligned} m_u/m_d &= 0.541 \pm 0.086 \text{ (0.51)}, \quad m_s/m_d = 18.9 \pm 1.6 \text{ (17.9)}, \\ m_c &= 0.73 \pm 0.17 \text{ (0.74)} \text{ GeV}, \quad m_s = 0.058 \pm 0.015 \text{ (0.046)} \text{ GeV}, \\ m_t &= 175 \pm 6 \text{ GeV}, \quad m_b = 2.91 \pm 0.07 \text{ GeV}, \end{aligned} \quad (71)$$

where the values in the parentheses are the theoretical values obtained from (68) for $m_t = 174$ GeV and $m_b = 2.9$ GeV. So, we see that the model can well reproduce the experimentally measured parameters.

The orthogonal matrices (62) are found to be

$$O_{uL} \simeq \begin{pmatrix} 0.9991 & -0.04252 & 1.269 \times 10^{-5} \\ 0.04244 & 0.9973 & 0.05964 \\ -2.548 \times 10^{-3} & -0.05958 & 0.9982 \end{pmatrix}, \quad (72)$$

$$O_{uR} \simeq \begin{pmatrix} -0.9991 & -0.04255 & -1.075 \times 10^{-5} \\ 0.04244 & -0.9966 & 0.07042 \\ -3.007 \times 10^{-3} & 0.07035 & 0.9975 \end{pmatrix}, \quad (73)$$

$$O_{dL} \simeq \begin{pmatrix} 0.9764 & 0.2160 & -1.856 \times 10^{-3} \\ -0.2159 & 0.9760 & 0.02899 \\ 8.074 \times 10^{-3} & -0.02790 & 0.9996 \end{pmatrix}, \quad (74)$$

$$O_{dR} \simeq \begin{pmatrix} -0.9695 & 0.2452 & 1.165 \times 10^{-4} \\ -0.2174 & -0.8599 & 0.4618 \\ 0.1133 & 0.4477 & 0.8870 \end{pmatrix}. \quad (75)$$

2. Lepton sector

The charged lepton mass matrix becomes

$$\mathbf{m}_e = \begin{pmatrix} -m_2 & m_2 & m_5 \\ m_2 & m_2 & m_5 \\ m_4 & m_4 & 0 \end{pmatrix} \exp(-i\theta_+^d), \quad (76)$$

where

$$m_2 = \frac{1}{2}Y_c^e v_+^d, \quad m_4 = \frac{1}{2}Y_{b'}^e v_+^d, \quad m_5 = \frac{1}{2}Y_b^e v_+^d. \quad (77)$$

The phase $\exp(-i\theta_+^d)$ can be rotated away, and all the mass parameters appearing in (76) are real. Diagonalization of the mass matrices is straightforward.

We would like to mention that the model has many predictions in this sector, because there are only four parameters to describe three light neutrino masses, three angles and three CP violating phases of V_{MNS} . Since the details of the predictions are presented in Refs. [23, 24, 54], we do not repeat them here again.⁸ Furthermore, the FCNC processes in the lepton sector have been very recently analyzed in details in Ref. [25], concluding that the model predictions of tree-level FCNC processes are at least five orders of magnitude smaller than the experimental upper bounds (The mass of the heavy neutral Higgs fields are

⁸ See also [55] for the predictions of the model on R parity violating processes. The leptonic sector of the present model is basically the same as the model of [23, 24], except for the spontaneous breaking of CP, which reduces one more independent phase in the leptonic sector.

assumed to be 120 GeV.) For instance, the branching fraction for $\mu \rightarrow e\gamma$ is seven orders of magnitude smaller than the expected experimental sensitivity [25]. Therefore, we shall not consider FCNCs in the leptonic sector in the following discussions.

B. CP violations and FCNCs in the SSB sector

If three generations of a family have the one+two structure, then the soft scalar mass matrices for the sfermions have a diagonal form (27):

$$\tilde{\mathbf{m}}_{aLL(RR)}^2 = m_{\tilde{a}}^2 \begin{pmatrix} a_{L(R)}^a & 0 & 0 \\ 0 & a_{L(R)}^a & 0 \\ 0 & 0 & b_{L(R)}^a \end{pmatrix} \quad (a = u, d, e), \quad (78)$$

where $m_{\tilde{a}}$ denote the average of the squark and slepton masses, respectively, and $(a_{L(R)}, b_{L(R)})$ are dimensionless free real parameters of $O(1)$. Because of the Q_6 flavor symmetry in the trilinear interactions, all the soft left-right mass matrices assume the form

$$(\tilde{\mathbf{m}}_{aLR}^2)_{ij} = A_{ij}^a (\mathbf{m}^a)_{ij} \quad (a = u, d, e), \quad (79)$$

where A_{ij}^a are free parameters of dimension one (see (26)). They are also real, because we impose CP invariance in the tri-linear couplings.

The quantities [56, 57]

$$\Delta_{LL(RR)}^a = U_{aL}^\dagger \tilde{\mathbf{m}}_{aLL(RR)}^2 U_{aL(R)} \quad \text{and} \quad \Delta_{LR}^a = U_{aL}^\dagger \tilde{\mathbf{m}}_{aLR}^2 U_{aR} \quad (80)$$

in the super CKM basis are used widely to parameterize FCNCs and CP violations coming from the SSB sector, where the unitary matrices U 's are given in (72)-(75).

1. CP violations

The imaginary parts of Δ 's (80) contribute to CP violating processes in the SSB sector. Recall that the soft scalar mass matrices $\mathbf{m}_{aLL,RR}^2$ are real, because they are diagonal, and that the phases of \mathbf{m}_{aLR}^2 come from the complex VEVs (32), because CP is only spontaneously broken in this sector. The unitary matrices U 's are complex, and so Δ 's can be complex, too. Note that the unitary matrices have the form $U = RPO$, where only P 's (given in (58)) are complex. Since P 's are diagonal, they commute with $\mathbf{m}_{aLL,RR}^2$, so that $\Delta_{LL,RR}^a$ have no imaginary part. Further \mathbf{m}_{aLR}^2 has the same phase structure as the corresponding fermion mass matrix \mathbf{m}^a , which can be made real according to (57)-(61). Therefore, Δ_{LR}^a , too, are real. Consequently, there is no CP violation originating from the SSB sector. The stringent constraints on Δ 's (80) coming from the EDMs [58, 59, 61] are automatically satisfied in this way of phase alignment.⁹

⁹ This does not mean that there is no CP violation in the SSB sector. Due to the existence of the multiple Higgs fields, there are one-loop diagrams contributing to the EDMs, even if all the SSB parameters are

2. FCNC

In Refs. [56, 57], [26]-[31] [58]-[61], experimental bounds on the dimensionless quantities

$$\delta_{LL,RR,LR}^a = \Delta_{LL,RR,LR}^a / m_a^2 \quad (a = u, d), \quad (81)$$

are given. The theoretical values of δ 's for the present model have been calculated in Ref. [22] as a function of the average sfermion masses and fine tuning parameters. The results may be summarized as follows. For the slepton sector where the average slepton mass $m_{\tilde{e}}$ is assumed to be 500 GeV, the theoretical values of $(\delta_{ij}^\ell)_{LL,RR,LR}$, except for $(\delta_{12}^\ell)_{LL}$, are several orders of magnitude smaller than the current experimental bounds, while $(\delta_{12}^\ell)_{LL}$ is of the same order as that of the experimental bound which comes from $\mu \rightarrow e\gamma$. In the squark sector, we find:

Up quark sector:

$$\begin{aligned} (\delta_{12}^u)_{LL} &= (\delta_{21}^u)_{LL} \simeq -1.5 \times 10^{-4} \Delta a_L^q, \\ (\delta_{12}^u)_{RR} &= (\delta_{21}^u)_{RR} \simeq -2.1 \times 10^{-4} \Delta a_R^u, \\ (\delta_{12}^u)_{LR} &\simeq -(\delta_{21}^u)_{LR} \simeq 6.2 \times 10^{-5} \left(-\tilde{A}_a^u + \tilde{A}_b^u + \tilde{A}_{b'}^u - \tilde{A}_c^u \right) \left(\frac{500 \text{ GeV}}{m_{\tilde{q}}} \right), \end{aligned} \quad (82)$$

Down quark sector

$$\begin{aligned} (\delta_{12}^d)_{LL} &= (\delta_{21}^d)_{LL} \simeq 2.2 \times 10^{-4} \Delta a_L^q, \\ (\delta_{13}^d)_{LL} &= (\delta_{31}^d)_{LL} \simeq -8.1 \times 10^{-3} \Delta a_L^q, \\ (\delta_{23}^d)_{LL} &= (\delta_{32}^d)_{LL} \simeq 2.8 \times 10^{-2} \Delta a_L^q, \\ (\delta_{12}^d)_{RR} &= (\delta_{21}^d)_{RR} \simeq -5.1 \times 10^{-2} \Delta a_R^d, \\ (\delta_{13}^d)_{RR} &= (\delta_{31}^d)_{RR} \simeq -0.1 \Delta a_R^d, \\ (\delta_{23}^d)_{RR} &= (\delta_{32}^d)_{RR} \simeq -0.4 \Delta a_R^d, \end{aligned} \quad (83)$$

where

$$\Delta a_L^q = a_L^q - b_L^q, \quad \Delta a_R^a = a_R^a - b_R^a, \quad \tilde{A}_i^a = \frac{A_i^a}{m_{\tilde{q}}} \quad (a = u, d). \quad (84)$$

These parameters, $a_{L,R}$ and \tilde{A}_i , are free dimensionless parameters, so that they are $O(1)$ if we do not fine tune them. The most stringent constraint in the up-sector comes from ΔM_D [30, 31]:

$$\Delta M_D \Rightarrow |(\delta_{12}^u)_{LL}|, |(\delta_{12}^u)_{RR}| \lesssim 6 \times 10^{-2}, \quad |(\delta_{12}^u)_{LR}|, |(\delta_{21}^u)_{LR}| \lesssim 10^{-2} \quad (85)$$

real. The diagrams typically contain the b terms, and we find that in the case of the present model $b_- \ll m_{H_-}^2$, $b_H < m_{\phi_H^{u,d}}^2$ (given in (50)) should be satisfied to satisfy the experimental constraints.

for $m_{\tilde{q}} = 0.5$ TeV. As we can see from (82) this constraint can be satisfied without a fine-tuning. As for the down-sector we have to satisfy the constraints coming from $\Delta M_K, \Delta M_{B_s}$ and ΔM_{B_d} [26, 29]:

$$\Delta M_K \Rightarrow |(\delta_{12}^d)_{LL}|, |(\delta_{12}^d)_{RR}|, |(\delta_{12}^d)_{LR}|, |(\delta_{21}^d)_{LR}| \lesssim 10^{-3} \quad (86)$$

$$\Delta M_{B_d} \Rightarrow |(\delta_{13}^d)_{LL}|, |(\delta_{13}^d)_{RR}|, |(\delta_{13}^d)_{LR}|, |(\delta_{31}^d)_{LR}| \lesssim 10^{-2} \quad (87)$$

$$\Delta M_{B_s} \Rightarrow |(\delta_{23}^d)_{LL}|, |(\delta_{23}^d)_{RR}|, |(\delta_{23}^d)_{LR}|, |(\delta_{32}^d)_{LR}| \lesssim 10^{-1} \quad (88)$$

Comparing these constraints with (83) we see that Δa_R^d should be fine tuned at the level of few %. ¹⁰ In the next subsections we assume that Δa_R^d is so small that only the heavy flavor-changing-neutral Higgs fields contribute to the mass differences of the neutral mesons.

C. Flavor changing neutral Higgs couplings

In Sec. IV we found that only the Higgs fields $\phi_{H,-}^{u,d}$ have flavor changing neutral couplings to the fermions, and that they have a definite form of mixing (see (49)). These are consequences of the Z_2 symmetry which is a part of the $O(2)$ flavor symmetry in the μ sector (as discussed in Sec. III. B). In the basis of the fermion mass eigenstates these Higgs couplings have the following form:

$$\begin{aligned} \mathcal{L}_{FCNC} = & - [Y_{ij}^{uH} \phi_H^u + Y_{ij}^{u-} \phi_-^u]^* \bar{u}'_{iL} u'_{jR} - [Y_{ij}^{dH} \phi_H^d + Y_{ij}^{d-} \phi_-^d]^* \bar{d}'_{iL} d'_{jR} \\ & - [Y_{ij}^{eH} \phi_H^d + Y_{ij}^{e-} \phi_-^d]^* \bar{e}'_{iL} e'_{jR} + h.c. , \end{aligned} \quad (89)$$

where the Higgs fields are defined in (43), and

$$\begin{aligned} \mathbf{Y}^{uH} &= U_L^{u\dagger} \left[\frac{1}{\sqrt{2}} \cos \gamma^u e^{-i\theta_+^u} (\mathbf{Y}^{u1} + \mathbf{Y}^{u2}) - \sin \gamma^u e^{-i\theta_3^u} \mathbf{Y}^{u3} \right] U_R^u, \\ &= O_L^{u\dagger} \left[\frac{1}{\sqrt{2}} \cos \gamma^u (\mathbf{Y}^{u1} + \mathbf{Y}^{u2}) - \sin \gamma^u \mathbf{Y}^{u3} \right] O_R^u, \end{aligned} \quad (90)$$

$$\begin{aligned} \mathbf{Y}^{dH} &= U_L^{d\dagger} \left[\frac{1}{\sqrt{2}} \cos \gamma^d e^{-i\theta_+^d} (\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) - \sin \gamma^d e^{-i\theta_3^d} \mathbf{Y}^{d3} \right] U_R^d, \\ &= O_L^{d\dagger} \left[\frac{1}{\sqrt{2}} \cos \gamma^d (\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) - \sin \gamma^d \mathbf{Y}^{d3} \right] O_R^d, \end{aligned} \quad (91)$$

$$\mathbf{Y}^{I-} = U_L^{I\dagger} \left[\frac{1}{\sqrt{2}} (\mathbf{Y}^{I1} - \mathbf{Y}^{I2}) \right] U_R^I \quad (I = u, d). \quad (92)$$

The Yukawa matrices \mathbf{Y}^{u1} etc. are given in (20), and the unitary matrices are given in (57)-(61) and (72)-(75).

¹⁰ We find that, as in the case of $(\delta_{12}^u)_{LR}$ of (82), the left-right insertions $|(\delta_{12,21,13,31,23,32}^d)_{LR}|$ are much smaller than these constraints.

The present model is consistent with the experimental observations in a certain region in the parameter space of the Yukawa couplings. An example of the choice of the nine parameters is given in (67), where we emphasize that this set of the nine parameters describe 10 physical independent quantities of the SM; six quark masses and four CKM parameters. Therefore, the consistent region in the space of the Yukawa couplings is very restricted, and we will be using only this set of the parameter values in the following discussion. Accordingly, for the values given in (67) we find the actual size of the Yukawa couplings:

$$Y_a^u = \frac{\sqrt{2}m_t y_u^2}{v_u \cos \gamma^u} \simeq \frac{0.9957}{\sin \beta \cos \gamma^u}, \quad Y_b^u = \frac{\sqrt{2}m_t b_u}{v_u \sin \gamma^u} \simeq \frac{0.05979}{\sin \beta \sin \gamma^u}, \quad (93)$$

$$Y_{b'}^u = \frac{\sqrt{2}m_t b'_u}{v_u \sin \gamma^u} \simeq \frac{0.07054}{\sin \beta \sin \gamma^u}, \quad Y_c^u = \frac{\sqrt{2}m_t q_u}{y_u v_u \cos \gamma^u} \simeq \frac{1.802 \times 10^{-4}}{\sin \beta \cos \gamma^u}, \quad (94)$$

$$Y_a^d = \frac{\sqrt{2}m_b y_d^2}{v_d \cos \gamma^d} \simeq \frac{0.01478}{\cos \beta \cos \gamma^d}, \quad Y_b^d = \frac{\sqrt{2}m_b b_d}{v_d \sin \gamma^d} \simeq \frac{5.449 \times 10^{-4}}{\cos \beta \sin \gamma^d}, \quad (95)$$

$$Y_{b'}^d = \frac{\sqrt{2}m_b b'_d}{v_d \sin \gamma^d} \simeq \frac{7.702 \times 10^{-3}}{\cos \beta \sin \gamma^d}, \quad Y_c^d = \frac{\sqrt{2}m_b q_d}{y_d v_d \cos \gamma^d} \simeq \frac{-6.701 \times 10^{-5}}{\cos \beta \cos \gamma^d}, \quad (96)$$

where γ 's and β are given in (44) and (48), respectively, and we have used: $m_t = 174$ GeV, $m_b = 2.9$ GeV and $v = \sqrt{v_u^2 + v_d^2} = 246$ GeV. These parameters are defined in the $\overline{\text{MS}}$ scheme and evaluated at the scale M_Z . With these numerical values we then obtain:

$$\mathbf{Y}^{uH} \simeq \frac{1}{\tan \gamma^u \sin \beta} \begin{pmatrix} -2.65 \times 10^{-4} & 3.22 \times 10^{-3} & 0.0439 \\ -3.22 \times 10^{-3} & 5.68 \times 10^{-3} & 0.0400 \\ 0.0519 & -0.0473 & 6.02 \times 10^{-3} \end{pmatrix} - \frac{\tan \gamma^u}{\sin \beta} \begin{pmatrix} 7.63 \times 10^{-6} & -3.58 \times 10^{-4} & -2.52 \times 10^{-3} \\ -1.54 \times 10^{-6} & -4.17 \times 10^{-3} & -0.0592 \\ -2.99 \times 10^{-3} & 0.0699 & 0.991 \end{pmatrix}, \quad (97)$$

$$\mathbf{Y}^{u-} \simeq \frac{\exp i(2\theta_3^u - \theta_+^u)}{\sin \gamma^u \sin \beta} \begin{pmatrix} 0 & -4.21 \times 10^{-3} & -0.0596 \\ -4.21 \times 10^{-3} & 0 & 2.54 \times 10^{-3} \\ 0.0704 & 3.00 \times 10^{-3} & 0 \end{pmatrix}, \quad (98)$$

$$\mathbf{Y}^{dH} \simeq \frac{1}{\tan \gamma^d \cos \beta} \begin{pmatrix} 6.63 \times 10^{-5} & 8.26 \times 10^{-5} & 2.80 \times 10^{-4} \\ -6.224 \times 10^{-5} & 3.74 \times 10^{-4} & 3.37 \times 10^{-4} \\ 4.10 \times 10^{-3} & -6.01 \times 10^{-3} & 2.52 \times 10^{-3} \end{pmatrix} - \frac{\tan \gamma^d}{\cos \beta} \begin{pmatrix} 1.37 \times 10^{-5} & 1.13 \times 10^{-4} & 7.56 \times 10^{-5} \\ 1.98 \times 10^{-5} & -1.88 \times 10^{-4} & -3.72 \times 10^{-4} \\ 1.67 \times 10^{-3} & 6.61 \times 10^{-3} & 0.0131 \end{pmatrix}, \quad (99)$$

$$\mathbf{Y}^{d-} \simeq \frac{\exp i(2\theta_3^d - \theta_+^d)}{\sin \gamma^d \cos \beta} \begin{pmatrix} 0 & -2.53 \times 10^{-4} & -4.72 \times 10^{-4} \\ -2.22 \times 10^{-4} & 0 & -1.04 \times 10^{-4} \\ 7.46 \times 10^{-3} & -1.89 \times 10^{-3} & 0 \end{pmatrix}. \quad (100)$$

The phases appearing in the matrices are given in (55) and (56). As we can see from these Yukawa matrices the size of the entries is fixed once the ratios of the VEVs ($\sin\beta, \sin\gamma^u$ etc.) are fixed. For the down-type Yukawa matrices (99) and (100), for instance, all the entries (except the (3,3) entry) are at most $O(10^{-3})$. All these facts originate from the flavor symmetries of the model. Needless to say that in multi-Higgs models without a flavor symmetry this situation is completely different.

D. FCNC

The most severe FCNC constraints on the theory come from the mass differences in the neutral meson systems; $\Delta M_D, \Delta M_K, \Delta M_{B_s}$ and ΔM_{B_d} .¹¹ The Yukawa interaction terms that contribute to them can be found from (89):

$$\begin{aligned}\mathcal{L}_{\Delta M_B} = & - [Y_{uc}^{uH} \phi_H^u + Y_{uc}^{u-} \phi_-^u]^* \bar{u}_L c_R - [Y_{cu}^{uH} \phi_H^u + Y_{cu}^{u-} \phi_-^u] \bar{u}_R c_L \\ & - [Y_{sd}^{dH} \phi_H^d + Y_{sd}^{d-} \phi_-^d]^* \bar{s}_L d_R - [Y_{ds}^{dH} \phi_H^d + Y_{ds}^{d-} \phi_-^d] \bar{s}_R b_L \\ & - [Y_{bd}^{dH} \phi_H^d + Y_{bd}^{d-} \phi_-^d]^* \bar{b}_L d_R - [Y_{db}^{dH} \phi_H^d + Y_{db}^{d-} \phi_-^d] \bar{b}_R d_L \\ & - [Y_{bs}^{dH} \phi_H^d + Y_{bs}^{d-} \phi_-^d]^* \bar{b}_L s_R - [Y_{sb}^{dH} \phi_H^d + Y_{sb}^{d-} \phi_-^d] \bar{b}_R s_L ,\end{aligned}\quad (101)$$

where the values of the Yukawa couplings can be read off from (97)-(100). In (101) we have dropped the prime on the fields, which was indicating the mass eigenstate. As we can see from (51), no $\phi - \phi$ and $\phi^* - \phi^*$ type propagators contribute to the mass differences. So, only the $\phi - \phi^*$ type propagators can contribute, implying the phases in the Yukawa couplings (101) cancel in the tree-level diagrams contributing to the mass differences.

The independent parameters entering into ΔM_D are:

$$\sin\beta, \sin\gamma^u, (M_H^u)^2 = \frac{(M_{H1}M_{H2})^2}{m_{\phi_H^d}^2}, (M_-^u)^2 = \frac{(M_{-1}M_{-2})^2}{m_{\phi_-^d}^2}, \quad (102)$$

where they are given, respectively, in (48), (44), (50), and (52). Similarly,

$$\cos\beta, \sin\gamma^d, (M_H^d)^2 = \frac{(M_{H1}M_{H2})^2}{m_{\phi_H^u}^2}, (M_-^d)^2 = \frac{(M_{-1}M_{-2})^2}{m_{\phi_-^u}^2} \quad (103)$$

enter into $\Delta M_K, \Delta M_{B_s}$ and ΔM_{B_d} . With these remarks in mind, we proceed.

D1: Constraint from ΔM_D

The following case is a special case of [31], and we will basically follow their analysis. An important difference here is that the size of all the Yukawa couplings is explicitly known.

¹¹ The contribution to ϵ'/ϵ is negligibly small, at most $O([10^{-7}/\alpha_S^2][\tilde{m}_q^2/M^2])$, where $\sim 10^{-7}$ originates from the Yukawa couplings relevant to this quantity, and \tilde{m}_q and M stand for the generic average squark and charged Higgs masses. See [62] and references therein for the constraint from the oblique corrections due to multiple $SU(2)_L$ doublet Higgs fields.

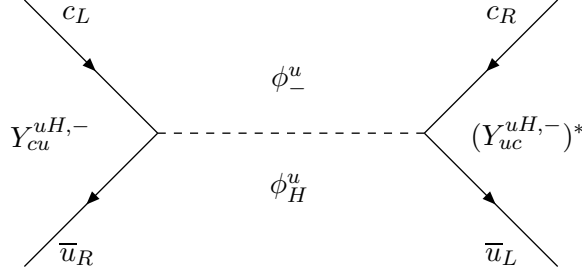


FIG. 1: The tree-diagram contributing to $(M_D^{\text{EXTRA}})_{12}$. Tree-diagrams contributing to M_K and $M_{B_{d,s}}$ are similar to this diagram. Leading QCD corrections [63] will be included, except for ΔM_K .

The tree-level diagrams are shown in Fig. 1. As we can see from Fig. 1, only the $\bar{u}_R c_L \bar{u}_L c_R$ type operator contributes to ΔM_D at the tree-level. The mass difference ΔM_D can then be obtained from

$$\Delta M_D = 2 |(M_D^{\text{SM}})_{12} + (M_D^{\text{EXTRA}})_{12}|, \quad (104)$$

where $(M_D^{\text{SM}})_{12}$ is the SM contribution, and

$$(M_D^{\text{EXTRA}})_{12} = 2C_D(\mu) \langle \bar{D}^0 | \bar{u}_R c_L \bar{u}_L c_R | D^0 \rangle (\mu), \quad (105)$$

$$C_D(\mu) = \eta(\mu) \left[\frac{Y_{cu}^{uH} (Y_{uc}^{uH})^*}{(M_H^u)^2} + \frac{Y_{cu}^{u-} (Y_{uc}^{u-})^*}{(M_-^u)^2} \right] \quad (106)$$

with the QCD correction $\eta(\mu)$. The operator $\bar{u}_R c_L \bar{u}_L c_R$ can mix with $\bar{u}_L \gamma^\mu c_L \bar{u}_R \gamma_\mu c_R$ even at the leading order in QCD in principle [64]. However, if $\bar{u}_L \gamma^\mu c_L \bar{u}_R \gamma_\mu c_R$ is absent at $\mu = \text{some energy}$, it will not be induced, at least in the leading order in QCD. Note that the values of the Yukawa matrices (97)-(100) are defined at $\mu = M_Z$, so that there are corrections if $\mu \neq M_Z$. We here take into account only QCD corrections because they are most dominant. The leading-order QCD correction η takes the form [64]

$$\eta(\mu_c = 2.8 \text{ GeV}) = \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_c)} \right]^{-24/25} \left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right]^{-24/23} \left[\frac{\alpha_s(M)}{\alpha_s(m_t)} \right]^{-8/7} \left[\frac{\alpha_s(M_Z)}{\alpha_s(M)} \right]^{-8/7} \quad (107)$$

$$\simeq 2.3, \quad (108)$$

where we have used the two-loop running of $\alpha_s(\mu)$ with $\alpha_s(M_Z) = 0.119$, and the last factor is the QCD correction to the Yukawa matrices. So, the M (which is supposed to be of the order of the heavy Higgs masses) dependence cancels nicely. The matrix element in the vacuum saturation approximation is given by [26]

$$\begin{aligned} \langle \bar{D}^0 | \bar{u}_R c_L \bar{u}_L c_R | D^0 \rangle (\mu_c = 2.8 \text{ GeV}) &= \frac{1}{4} f_D^2 B'_D M_D \left(\frac{M_D}{m_c} \right)^2 \\ &\simeq 3.1 \times 10^{-2} \text{ GeV}^3, \end{aligned} \quad (109)$$

| Input | | Input | |
|---------------------|--|-------------------------------|--|
| f_D | $(222.6 \pm 16.7 \begin{smallmatrix} +2.8 \\ -3.4 \end{smallmatrix}) \times 10^{-3} \text{ GeV}$ | $B'_D(2.8\text{GeV})$ | 1.08 ± 0.03 |
| M_D | $1.8645 \pm 0.0004 \text{ GeV}$ | τ_D | $(410.1 \pm 1.5) \times 10^{-3} \text{ ps}$ |
| x_D | $(5.3 - 11.7) \times 10^{-3}$ | f_K | $(159.8 \pm 1.4 \pm 0.44) \times 10^{-3} \text{ GeV}$ |
| f_{B_s} | I: 0.240 ± 0.040 II: 0.245 ± 0.013 GeV | $B'_s(m_b)$ | $1.16 \pm 0.02 \begin{smallmatrix} +0.05 \\ -0.07 \end{smallmatrix}$ |
| $f_{B_s}\sqrt{B_s}$ | I: 0.221 ± 0.046 II: 0.227 ± 0.017 GeV | ξ | 1.24 ± 0.04 |
| f_{B_d} | $0.198 \pm 0.017 \text{ GeV}$ | $B'_d(m_b)$ | $1.15 \pm 0.03 \begin{smallmatrix} +0.05 \\ -0.07 \end{smallmatrix}$ |
| M_K | $0.497648 \pm 0.000022 \text{ GeV}$ | ΔM_K^{exp} | $(0.5292 \pm 0.0009) \times 10^{-2} \text{ ps}^{-1}$ |
| M_{B_s} | $5.3661 \pm 0.0006 \text{ GeV}$ | $\Delta M_{B_s}^{\text{exp}}$ | $17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1}$ |
| M_{B_d} | $5.27950 \pm 0.00033 \text{ GeV}$ | $\Delta M_{B_d}^{\text{exp}}$ | $0.507 \pm 0.005 \text{ ps}^{-1}$ |
| $m_u(2\text{GeV})$ | $(3 \pm 1) \times 10^{-3} \text{ GeV}$ | $m_c(m_c)$ | $1.30 \pm 0.05 \text{ GeV}$ |
| $m_d(2\text{GeV})$ | $(6.0 \pm 1.5) \times 10^{-3} \text{ GeV}$ | $m_s(2\text{GeV})$ | $0.10 \pm 0.02 \text{ GeV}$ |
| $m_d(m_b)$ | $(5.1 \pm 1.3) \times 10^{-3} \text{ GeV}$ | $m_s(m_b)$ | $0.085 \pm 0.017 \text{ GeV}$ |
| $m_t(m_t)$ | $163.8 \pm 2.0 \text{ GeV}$ | $m_b(m_b)$ | $4.22 \pm 0.08 \text{ GeV}$ |

TABLE III: Parameter values used in the text (see also Ref. [52]). f_D is taken from [65], and we use B'_D and x_D of [30] and [66], respectively. $M_D, \tau_D, f_K, M_K, \Delta M_K^{\text{exp}}, M_{B_s}, M_{B_d}, \Delta M_{B_d}^{\text{exp}}$ are from [51]. f_{B_s} (I) and $f_{B_s}\sqrt{B_s}$ (I) are the conservative sets of [72], and $f_{B_s}\sqrt{B_s}$ (II) is found in [67], while f_{B_s} (II) and ξ are taken from [68], and f_{B_d} is obtained from f_{B_s}/ξ . (See [69] for a more conservative estimate of ξ , and references therein.) B'_s and B'_d are found in [70]. $\Delta M_{B_s}^{\text{exp}}$ is from [71]. $m_u(2\text{GeV})$ and $m_d(2\text{GeV})$ are from [51], while the mass values of the other quarks are taken from [72], in which the relevant references are given.

where we have used the central values of the parameters¹² given in Table III. ($m_c(2.8 \text{ GeV}) = 1.0 \text{ GeV}$ which corresponds to $m_c(m_c) = 1.3 \text{ GeV}$.)

Clearly, the larger $(M_H^u)^2$ and $(M_-^u)^2$ are, the smaller are the extra contributions. Here we are interested in the minimal values of $(M_H^d)^2$ and $(M_-^d)^2$, which are consistent with the observations. We find that the Wilson coefficient C_D becomes

$$\begin{aligned}
C_D(\mu_c) &= \frac{\eta(\mu_c)}{\sin^2 \beta} \left[\frac{1 \text{ TeV}}{M_H^u} \right]^2 \times 10^{-11} \\
&\times \left(\frac{1.772}{r_u^2 \sin^2 \gamma^u} - \frac{1.037}{\tan^2 \gamma^u} - 0.115 + 5.5 \times 10^{-5} \tan^2 \gamma^u \right) \text{ GeV}^{-2}, \quad (110)
\end{aligned}$$

¹² Since we take here a conservative standpoint that the extra contribution can be as large as the experimental value, we ignore the details of uncertainties.

where

$$r_u = \frac{M_-^u}{M_H^u} = \left(\frac{M_{-1} M_{-2}}{M_{H1} M_{H2}} \right) \left(\frac{m_{\phi_H^d}}{m_{\phi_-^d}} \right), \quad (111)$$

and the mass parameters are defined in (102). If each term in (110) should satisfy the constraint,

$$|\Delta M_D^{\text{EXTRA}}| = 2 |(M_D^{\text{EXTRA}})_{12}| < \Delta M_D^{\text{exp}} = x_D / \tau_D \simeq 1.4 \times 10^{-14} \text{ GeV}, \quad (112)$$

one finds that $\sin \beta M_H^u \gtrsim 17 \text{ TeV}$ and $\sin \beta M_-^u \gtrsim 22 \text{ TeV}$ should be satisfied. We, however, observe that the terms in (110) can cancel each other, so that no lower bounds on M_H^u and M_-^u can be obtained. In Fig. 2 we show the region in the $\sin \gamma^u - r_u$ plane for $\sin \beta M_H^u = 2 \text{ TeV}$, in which $|\Delta M_D^{\text{EXTRA}}|$ is smaller than the smallest ΔM_D^{exp} , i.e.,

$$|\Delta M_D^{\text{EXTRA}}| < 8 \times 10^{-15} \text{ GeV}. \quad (113)$$

We see from Fig. 2 that to satisfy the constraint (113), we have to fine tune r_u and $\sin \gamma^u$

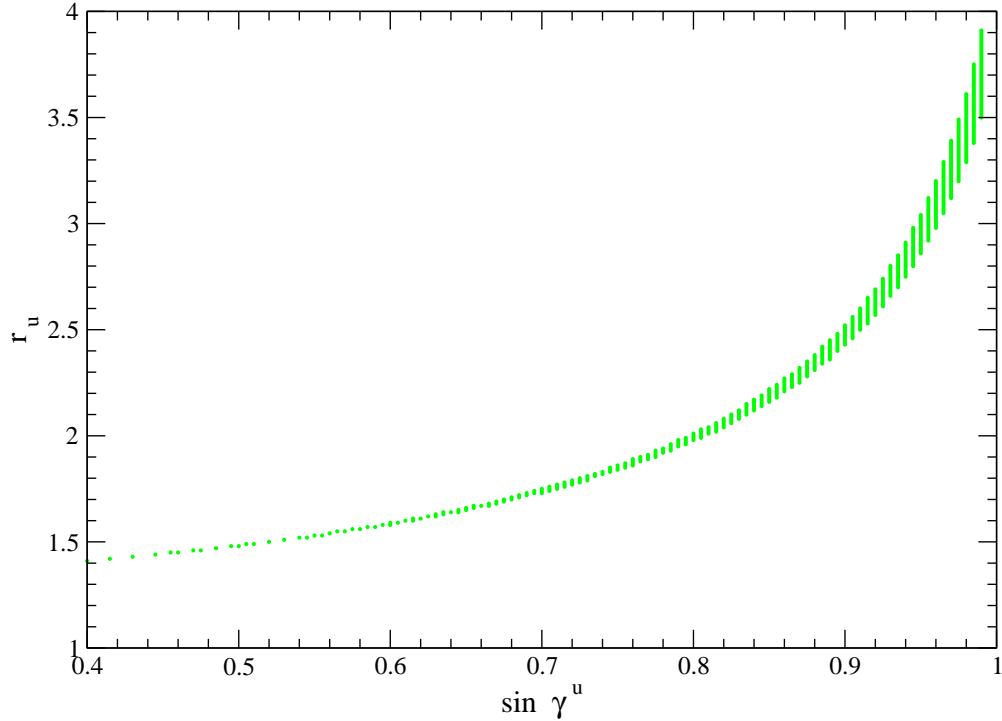


FIG. 2: The region in the $\sin \gamma^u - r_u$ plane, in which the constraint (113) coming from ΔM_D is satisfied for $\sin \beta M_H^u = 2 \text{ TeV}$, where r_u , $\sin \gamma^u$ and M_H^u are defined in (111), (44) and (102), respectively.

even for $\sin \beta M_H^u = 2 \text{ TeV}$.

The neutral Higgs bosons in question can induce processes such as $D^0 \rightarrow e^+ e^-$ and $D^0 \rightarrow \mu^+ \mu^-$ which are strongly suppressed. The experimental upper bounds of the branching ratios

are smaller than $O(10^{-6})$. From a rough estimate we find that $M_-^u, M_H^u > M_Z$ is more than sufficient to suppress these processes. So, in principle, M_-^u, M_H^u could be light, although one needs an extreme fine tuning between r_u and $\sin \gamma^u$.

D2: Constraint from ΔM_K

As in the case of ΔM_D , the interaction Lagrangian generates only one type of the $\Delta S = 2$ operator at the tree level. So, the relevant matrix element is

$$\begin{aligned} \langle \bar{K}^0 | \bar{s}_R^\alpha d_L^\alpha \bar{s}_L^\beta d_R^\beta | K^0 \rangle &= \frac{1}{4} f_{B_K}^2 B'_K M_K \left(\frac{M_K}{m_s + m_d} \right)^2 \\ &\simeq 0.28 \text{ GeV}^3, \end{aligned} \quad (114)$$

where we have used the central values of the parameters given in Table III. (As in the case of ΔM_D we ignore the details of uncertainties involved in ΔM_D .) As far as we understand, there is no reliable calculation of B'_K for the present case (114),¹³ and so we assume that $B'_K = 1$. Correspondingly, we do not take into account QCD corrections for the present case.

The tree-level coefficient is given by

$$\begin{aligned} C_K &= \left[\frac{Y_{ds}^{dH} (Y_{sd}^{dH})^*}{(M_H^d)^2} + \frac{Y_{ds}^{d-} (Y_{sd}^{d-})^*}{(M_-^d)^2} \right] \\ &= \frac{1}{\cos^2 \beta} \left[\frac{1 \text{ TeV}}{M_H^d} \right]^2 \times 10^{-14} \\ &\times \left(\frac{5.617}{r_d^2 \sin^2 \gamma^d} - \frac{0.514}{\tan^2 \gamma^d} - 0.539 + 0.224 \tan^2 \gamma^d \right) \text{ GeV}^{-2}, \end{aligned} \quad (115)$$

where

$$r_d = \frac{M_-^d}{M_H^d} = \left(\frac{M_{-1} M_{-2}}{M_{H1} M_{H2}} \right) \left(\frac{m_{\phi_H^d}}{m_{\phi_-^d}} \right). \quad (116)$$

In Fig. 3 we show the region in the $r_d - \sin \gamma^d$ plane in which

$$\Delta M_K = 2 \times 0.28 \times C_K \text{ GeV} < \Delta M_K^{\text{exp}} \simeq 3.49 \times 10^{-15} \text{ GeV} \quad (117)$$

is satisfied.

D3: Constraint from $\Delta M_{B_s}, \Delta M_{B_d}$

As in the previous cases, the mass differences can be obtained from

$$\Delta M_{B_{s,d}} = 2 \left| \langle \bar{B}^0 | (M_{s,d}^{\text{SM}})_{12} + (M_{s,d}^{\text{EXTRA}})_{12} | B^0 \rangle \right|. \quad (118)$$

¹³ See [73] for a lattice calculation of B'_K of the present case, and also comments of [74].

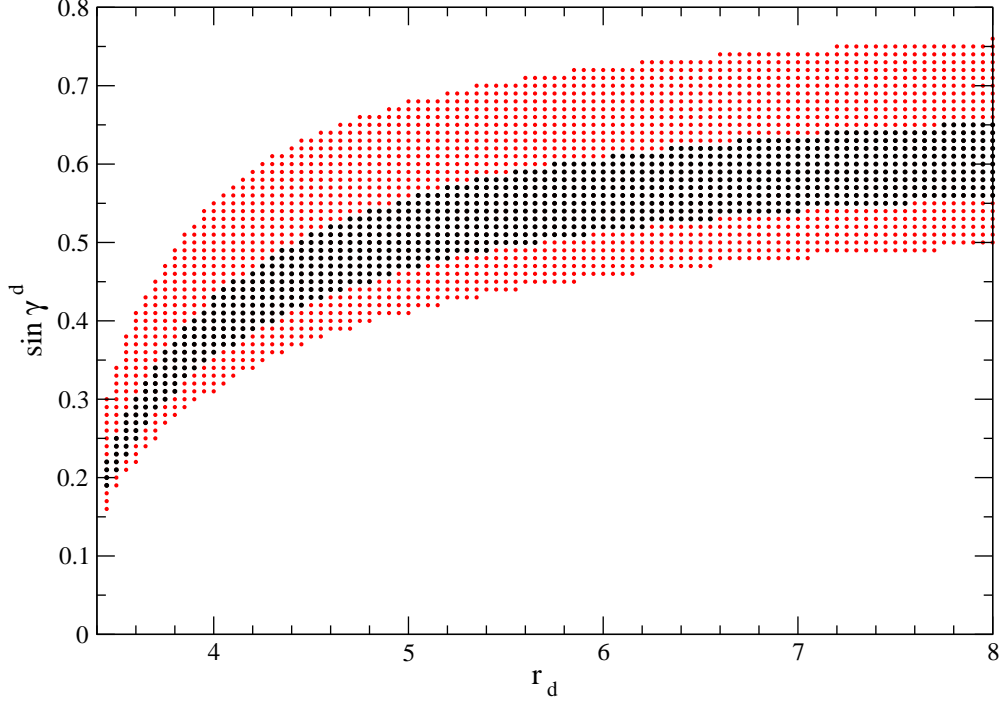


FIG. 3: The region in the $r_d - \sin \gamma^d$ plane for $\cos \beta M_H^d = 0.5$ TeV (red (dark grey)) and 0.3 TeV (black), in which $|\Delta M_K^{\text{EXTRA}}| < \Delta M_K^{\text{exp}}$ is satisfied. r_d and $\sin \gamma^d$ are defined in (116) and (44), respectively.

The SM contributions to $\Delta M_{B_s}, \Delta M_{B_d}$ are well controlled up to the numerical uncertainty in the decay constants. Here following [72], which is based on the NLO-QCD calculations in Refs. [75] and [76], we consider two sets of the uncertainties for the B system, I and II, as one can see in Table III. Since the uncertainties in the decay constants are much larger than those of other quantities, we assume that

$$f_{B_s} \sqrt{B_s} = \begin{cases} 0.221 \pm 0.046 & \text{for the parameter set I} \\ 0.227 \pm 0.017 & \text{for the parameter set II} \end{cases}, \quad (119)$$

$$f_{B_d} \sqrt{B_d} = \begin{cases} 0.181 \pm 0.044 & \text{for the parameter set I} \\ 0.184 \pm 0.017 & \text{for the parameter set II} \end{cases} \quad (120)$$

are the only uncertainties for the SM model contributions $M_{s,d}^{\text{SM}}$, where $f_{B_d} \sqrt{B_d}$ is obtained from $\xi = f_{B_s} \sqrt{B_s} / f_{B_d} \sqrt{B_d}$. To simplify the situation further, we assume that this is also true for the extra contributions $M_{s,d}^{\text{EXTRA}}$.

To calculate $(M_{s,d}^{\text{SM}})_{12}$ we use the parameter values (68) which are predicted in the present model:

$$|V_{\text{CKM}}|_{us} = 0.2266, |V_{\text{CKM}}|_{ub} = 0.00362, |V_{\text{CKM}}|_{cb} = 0.0417, \phi_3(\gamma) = 1.107. \quad (121)$$

Then we follow the calculation of [72] and obtain:

$$2(M_{B_s}^{\text{SM}})_{12} = 2 |(\bar{M}_s^{\text{SM}})_{12}| (1 \pm \delta_s) \exp i\phi_s$$

$$= \begin{cases} 19.5(1 \pm 0.46) \exp(-i0.0035) \\ 20.6(1 \pm 0.16) \exp(-i0.0035) \end{cases} \text{ ps}^{-1} \text{ for } \begin{cases} \text{I} \\ \text{II} \end{cases}, \quad (122)$$

$$2(M_{B_s}^{\text{SM}})_{12} = 2 |(\bar{M}_d^{\text{SM}})_{12}| (1 \pm \delta_d) \exp i\phi_d$$

$$= \begin{cases} 0.56(1 \pm 0.55) \exp(i0.77) \\ 0.59(1 \pm 0.19) \exp(i0.77) \end{cases} \text{ ps}^{-1} \text{ for } \begin{cases} \text{I} \\ \text{II} \end{cases}, \quad (123)$$

where $(\bar{M}_{s,d}^{\text{SM}})_{12}$ are the SM contributions which are obtained with the central values of $f_{B_s}\sqrt{B_s}, \xi, M_{B_{s,d}}$ and the quark masses given ¹⁴ in III and $\alpha_s(M_Z) = 0.119$, and δ_s and δ_d correspond to the uncertainties in $f_{B_s}\sqrt{B_s}$ and $f_{B_d}\sqrt{B_d}$ given in (120), respectively. As we can see from Table III, the SM values are slightly larger than the experimental values.

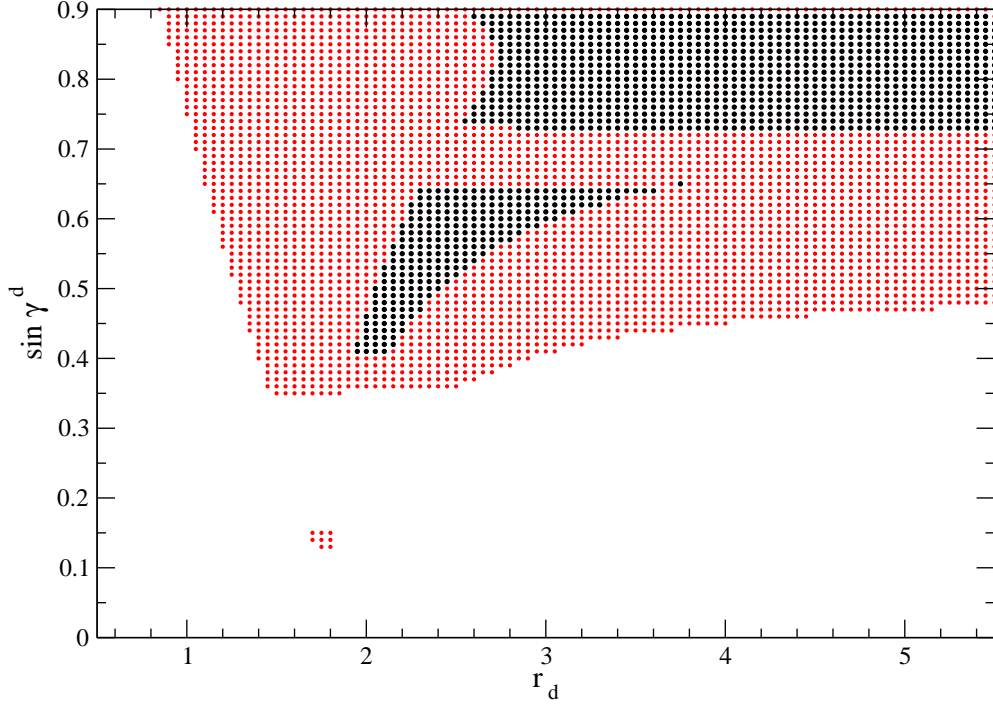


FIG. 4: The allowed region for the parameter set I with $\cos \beta M_H^d = 0.50$ (black) and 1.5 (red (dark grey)) TeV in which the constraints (129) and (130) are simultaneously satisfied. r_d and $\sin \gamma^d$ are defined in (116) and (44), respectively. Two sets of values I and II are given in Table III.

¹⁴ The model does not predict the absolute scale for the quark masses. If we use the mass ratio given in (68), we obtain a slightly smaller value for $m_b(m_b)$ (while we obtain the same value for $m_c(m_c)$). This difference has only a negligible effect on the SM contributions.

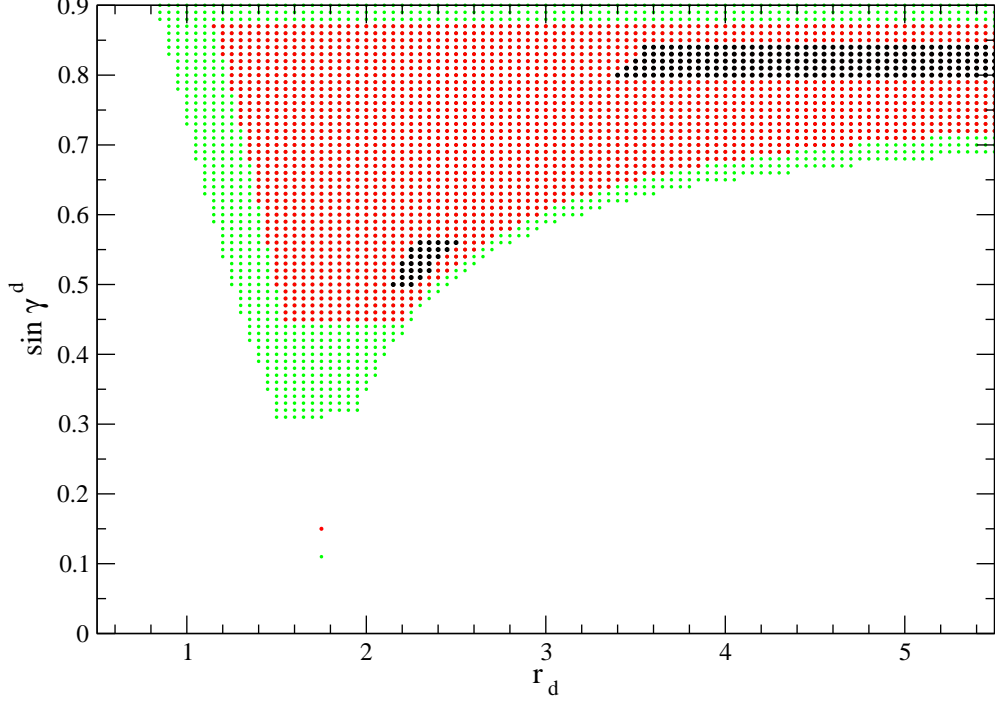


FIG. 5: The same as Fig. 4 for the parameter set II with $\cos \beta M_H^d = 0.50$ (black), 1.5 (red (dark grey)) and 2 (green (grey)) TeV .

As for the extra contributions, only the matrix elements

$$\begin{aligned}
 \langle \overline{B}_s^0 | \overline{b}_R^\alpha s_L^\alpha \overline{b}_L^\beta s_R^\beta | B_s^0 \rangle &= \frac{1}{4} f_{B_s}^2 B'_s M_{B_s} \left(\frac{M_{B_s}}{m_b + m_s} \right)^2 \\
 &\simeq \begin{cases} 0.29 \text{ (I)} \\ 0.30 \text{ (II)} \end{cases} \text{ GeV}^3
 \end{aligned} \tag{124}$$

and

$$\begin{aligned}
 \langle \overline{B}_d^0 | \overline{b}_R^\alpha d_L^\alpha \overline{b}_L^\beta d_R^\beta | B_d^0 \rangle &= \frac{1}{4} f_{B_d}^2 B'_d M_{B_d} \left(\frac{M_{B_d}}{m_b + m_d} \right)^2 \\
 &\simeq 0.18 \text{ (I,II)} \text{ GeV}^3
 \end{aligned} \tag{125}$$

are relevant for $\Delta M_{B_s}, \Delta M_{B_d}$, where the tree-level diagrams similar to Fig. 1 contribute to these mass differences, and we have used the central values of the parameters in Table III.

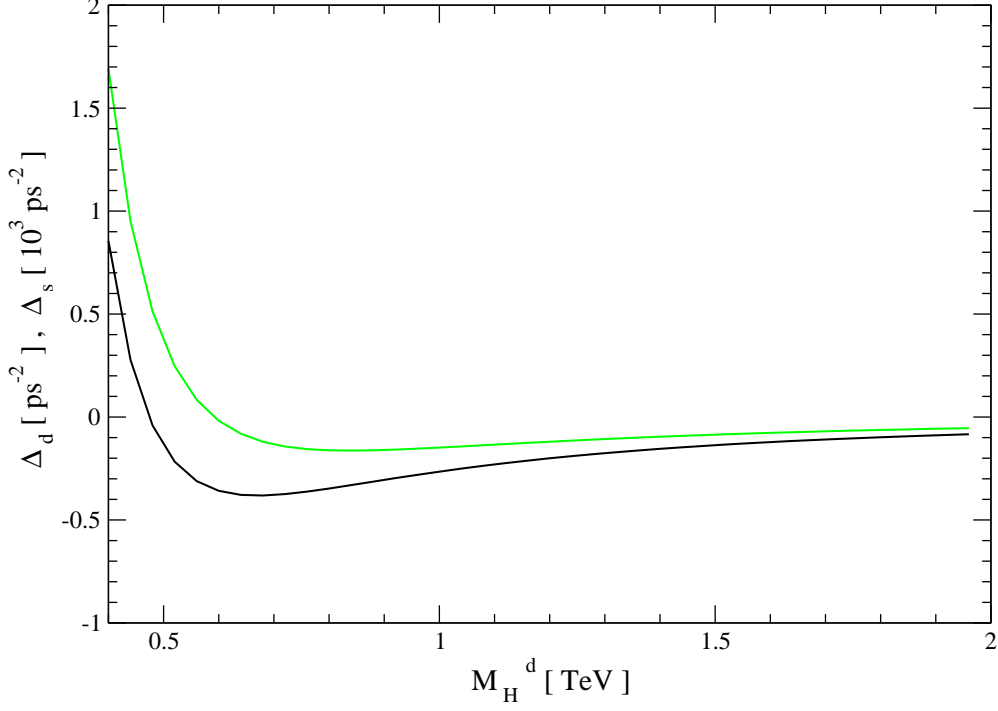


FIG. 6: Δ_s (green (grey)) and Δ_d (black) for the parameter set I with $r_d = 3$ and $\sin \gamma^d = 0.8$, where they are defined in (132). This graph explains why the allowed region in the $r_d - \sin \gamma^d$ plane first shrinks and then extends as M_H^d increases. For the the parameter set II we obtain a similar result.

The leading order Wilson coefficients are

$$C_{B_s} = \eta_B(m_b) \frac{1}{\cos^2 \beta} \left[\frac{1 \text{ TeV}}{M_H^d} \right]^2 \times 10^{-12} \times \left(\frac{0.197}{r_d^2 \sin^2 \gamma^d} - \frac{2.025}{\tan^2 \gamma^d} - 4.463 - 2.459 \tan^2 \gamma^d \right) \text{ GeV}^{-2}, \quad (126)$$

$$C_{B_d} = \eta_B(m_b) \frac{1}{\cos^2 \beta} \left[\frac{1 \text{ TeV}}{M_H^d} \right]^2 \times 10^{-12} \times \left(-\frac{3.521}{r_d^2 \sin^2 \gamma^d} + \frac{1.148}{\tan^2 \gamma^d} - 0.780 + 0.127 \tan^2 \gamma^d \right) \text{ GeV}^{-2}, \quad (127)$$

where

$$\eta_B(m_b = 4.22 \text{ GeV}) = \left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right]^{-24/23} \left[\frac{\alpha_s(M)}{\alpha_s(m_t)} \right]^{-8/7} \left[\frac{\alpha_s(M_Z)}{\alpha_s(M)} \right]^{-8/7} \simeq 2.0. \quad (128)$$

Then we require that

$$\Delta M_{B_s} = \Delta M_{B_s}^{\text{exp}} = 2 \left| (M_{B_s}^{\text{SM}})_{12} + 0.41 \times C_{B_s} \right| = 17.77 \text{ ps}^{-1} = 1.17 \times 10^{-11} \text{ GeV}, \quad (129)$$

$$\Delta M_{B_d} = \Delta M_{B_d}^{\text{exp}} = 2 \left| (M_{B_d}^{\text{SM}})_{12} + 0.25 \times C_{B_d} \right| = 0.507 \text{ ps}^{-1} = 3.34 \times 10^{-13} \text{ GeV}. \quad (130)$$

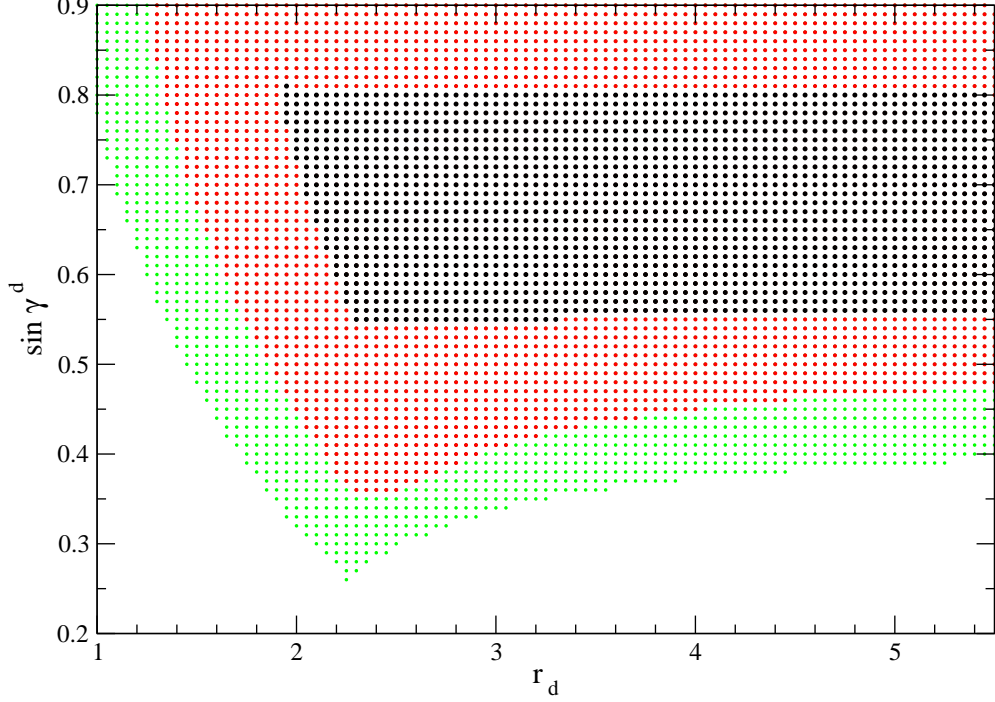


FIG. 7: The region in which the constraints (117), (129) and (130) coming from $\Delta M_K, \Delta M_{B_{s,d}}$ are satisfied for the parameter set I with $M_H^d = 1.1$ (black), $M_H^d = 1.5$ (red (dark grey)) and 2 (green (grey)) TeV. r_d and $\sin \gamma^d$ are defined in (116) and (44), respectively.

Note that according to our assumption the uncertainties factorize as $(1 \pm \delta_{s,d})[(\bar{M}_{s,d}^{\text{SM}})_{12} + (M_{s,d}^{\text{EXTRA}})_{12}]$, where $\bar{M}_{s,d}$ are the central values and $\delta_{s(d)}$ are given in (122) and (123). In Fig. 4 and 5 we show the allowed region in the $r_d - \sin \gamma^d$ plane for the parameter sets I and II, respectively, in which (129) and (130) are satisfied. We find that (129) and (130) can be simultaneously satisfied even for small $M_H^d \gtrsim 0.50$ TeV. The allowed region shrinks as M_H^d increases. At $M_H^d = 1$ TeV the allowed region is very small. But a wide allowed region exists for $M_H^d = 2$ TeV. The reason that the allowed region first decreases and then increases as M_H^d increases starting from $\simeq 0.50$ TeV is the following. The constraint (129) and (130) can be written as

$$[\Delta M_{B_{s,d}}^{\text{exp}}]^2 (1 + \delta_s)^{-2} \leq \Delta_{s,d} + 4[(\bar{M}_s^{\text{SM}})_{12}]^2 \leq [\Delta M_{B_{s,d}}^{\text{exp}}]^2 (1 - \delta_s)^{-2}, \quad (131)$$

where

$$\Delta_{s,d} = 4[(M_{s,d}^{\text{EXTRA}})_{12}]^2 + 8 \cos \phi_{s,d} (\bar{M}_{s,d}^{\text{SM}})_{12} (M_{s,d}^{\text{EXTRA}})_{12}, \quad (132)$$

and $(\bar{M}_{s,d}^{\text{SM}})_{12}$ and $\phi_{s,d}$ are given in (122) and (123). For a large M_H^d the second term of $\Delta_{s,d}$ is dominant. However, for a small M_H^d , two terms can become of the same order, and since $(M_{B_s}^{\text{EXTRA}})_{12}$ and $(M_{B_d}^{\text{EXTRA}})_{12}$ can simultaneously become negative, these two terms can cancel each other, so that the both constraints (131) for ΔM_{B_s} and ΔM_{B_d} can be

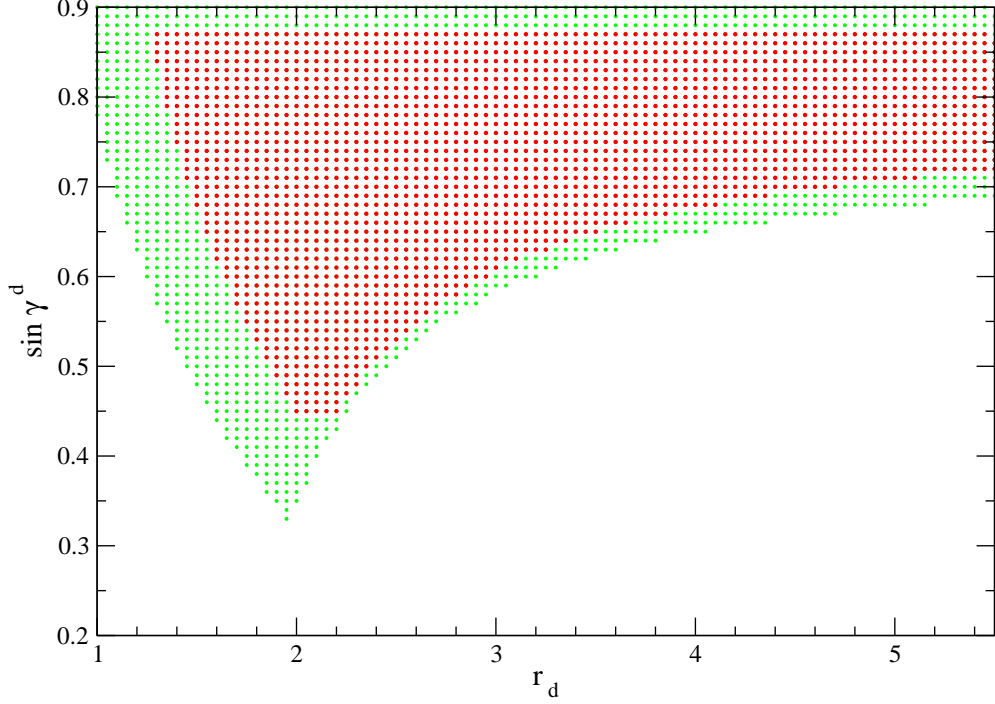


FIG. 8: The same as Fig. 7 for the parameter set II with $M_H^d = 1.5$ (red (dark grey)) and 2 (green (grey)) TeV.

simultaneously satisfied. In Fig. 6 we show Δ_s (red) and Δ_d (blue) for the parameter set I as a function of M_H^d for $r_d = 3$ and $\sin \gamma^d = 0.8$, where we vary M_H^d from 0.4 TeV to 2 TeV. We see from the figure that Δ_s and Δ_d decrease as M_H^d increases for $M_H^d \lesssim 0.6$ TeV. Note that the constraint from ΔM_{B_d} (130) is stronger than that from ΔM_{B_s} (129). In this region the constraint from Δ_K is not satisfied. But if we relax the constraint (because non-perturbative contributions to Δ_K suffer from large uncertainties) to $\Delta M_K^{\text{EXTRA}} < 2\Delta M_K^{\text{exp}}$, then it is satisfied.

Next we consider the region in which all the three constraints (117), (129) and (130) are satisfied. We find that the small M_H^d region in Fig. 4 and 5 disappears, and that $M_H^d \gtrsim 1.0$ (I) and 1.3 (II) TeV have to be satisfied. In Fig. 7 and 8 we show the allowed region in which all the constraints (117), (129) and (130) are satisfied for $M_H^d = 1.5$ TeV (blue) $M_H^d = 2$ TeV (green).

D4: Constraint from $\Delta M_{B_s}/\Delta M_{B_d}$

This ratio is important to determine experimentally $|V_{td}/V_{ts}|$. This is true only if there is no other contribution than the SM ones. In the presence of the extra neutral Higgs bosons, the situation changes. Here we ask ourselves how heavy the extra extra neutral Higgs bosons should be, or where the allowed region in the $r_d - \sin \gamma^d$ plane for a given $\cos \beta M_H^d$ is, such that the determination of $|V_{td}/V_{ts}|$ from the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ is not influenced.

The largest theoretical uncertainty in the mass ratio is contained in $\xi =$

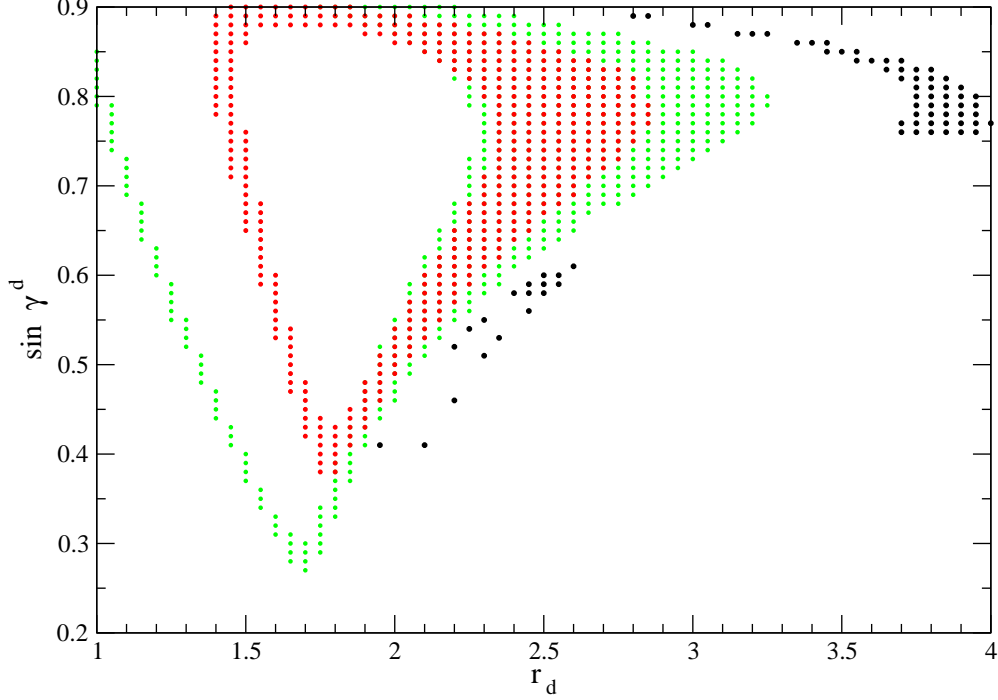


FIG. 9: The allowed region for the parameter set I with $\cos \beta M_H^d = 0.50$ (black), 1.5 (red (dark grey)) and 2 (green (grey)) TeV, in which the constraints (129), (130) and (133) coming from $\Delta M_{B_{s,d}}$ and $\Delta M_{B_s}/\Delta M_{B_d}$ are simultaneously satisfied.

$f_{B_s}\sqrt{B_s}/f_{B_d}\sqrt{B_d} = 1.24 \pm 0.04$ (see Table III), that is, 3.2% uncertainty, which is larger than the experimental ones. Accordingly, we require that the theoretical value of $\Delta M_{B_s}/\Delta M_{B_d}$ should be equal to the experimental central value 35.05 within an error of 5% (the mass ratio is proportional to ξ^2), i.e.

$$\Delta M_{B_s}/\Delta M_{B_d} = 35.05(1 \pm 0.05) . \quad (133)$$

We require that (129), (130) and (133) are simultaneously satisfied. The allowed region is shown in Fig. 9 and 10 for $M_H^d = 0.50$ (red), 1.5 (blue) and 2 (green) TeV. We see that the small M_H^d region of Fig. 4 and 5 is still there. We also find that $M_H^d \gtrsim 1.1$ (1.3) for the parameter set I (II) TeV or $0.39 \text{ TeV} \lesssim M_H^d \lesssim 0.65$ ($0.45 \text{ TeV} \lesssim M_H^d \lesssim 0.6$) TeV for the parameter set I (II) if $\Delta M_{B_s}/\Delta M_{B_d}$ is equal to the experimental central value 35.05 within an error of 1%.

VI. CONCLUSION

We have considered a supersymmetric extension of the SM based on the discrete Q_6 family symmetry, which has been recently proposed in Refs. [7, 21, 22]. We have stressed the one + two structure for each family; one Q_6 singlet and one Q_6 doublet for each family including the $SU(2)_L$ doublet Higgs fields. We have found that it is possible to realize the

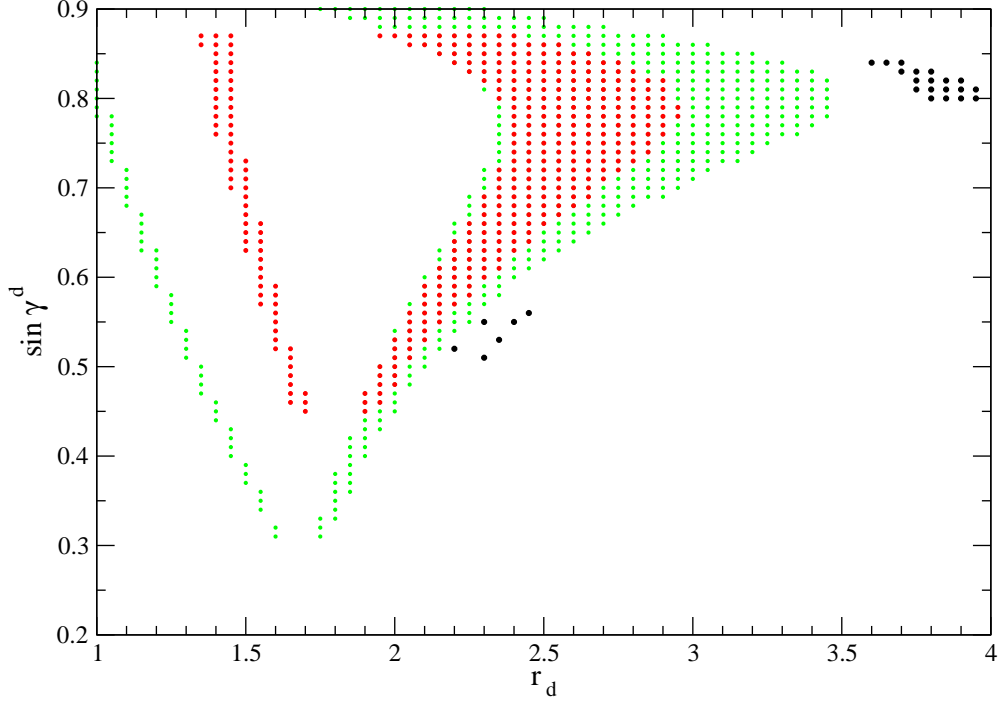


FIG. 10: The same as Fig. 9 for the parameter set II. Two set of values I and II are given in Table III.

one + two structure in a renormalizable way, so that the Higgs sector becomes minimal and much simpler than that of the original model of [7, 21, 22]. In this way the Higgs sector can be investigated with much less assumptions. It is explicitly shown that the SSB parameters can be fine tuned so as to make the heavy Higgs bosons much heavier than M_Z and at the same time to obtain a desired size of spontaneous CP violation to reproduce the Kobayashi-Maskawa CP violating phase.

We have investigated the FCNC processes, especially those mediated by heavy neutral Higgs bosons. Because of the Q_6 family symmetry, the number of the independent Yukawa couplings is smaller than that of the observed quantities such as the CKM matrix and the quark masses. Therefore, the FCNCs can be parametrized only by the mixing angles and masses of the Higgs fields: There are two angles and four mass parameters that enter into the FCNCs for a given $\tan \beta$; a set of three parameters for ΔM_D and another set of three parameters for ΔM_K and $\Delta M_{B_{d,s}}$. We have expressed the mass differences of the neutral mesons ΔM_K , ΔM_D and $\Delta M_{B_{d,s}}$ in terms of these parameters.

Since the SM contributions to ΔM_{B_s} and ΔM_{B_d} are well-controlled, we haven taken into account them to obtain the constraints from ΔM_{B_s} and ΔM_{B_d} . That is, we have assumed that the extra contributions are allowed only in a small window in which the SM values differ from the experimental values. Allowed ranges in which the constraints are satisfied are shown in various figures, where ΔM_K , ΔM_{B_s} and ΔM_{B_d} take values in the common parameter space. We have also investigated the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ in the region, in which all the constraints from ΔM_{B_s} and ΔM_{B_d} are simultaneously satisfied, and found that in

a wide subregion the ratio differs from the experimental central value only by less than 5%. If we require that all the constraints from ΔM_K , ΔM_{B_s} and ΔM_{B_d} including the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ are satisfied, we have found that the heavy Higgs bosons should be heavier than ~ 1.5 TeV. If we relax the constraint from ΔM_K to $\Delta M_K^{\text{EXTRA}} < 2\Delta M_K^{\text{exp}}$ (because of the reason that non-perturbative contributions suffer from large uncertainties), the heavy Higgs bosons can be as light as ~ 0.4 TeV, which is within the accessible range of LHC [5].

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- [1] G. Altarelli, Lectures given at the Summer Institute 2007, Fuji-Yoshida, Japan, 3 -10 Aug. 2007, and arXiv:0711.0161 [hep-ph].
 - [2] E. Ma, arXiv:0705.0327 [hep-ph].
 - [3] A. Mondragon, AIP Conf. Proc. **857B** (2006) 266 [arXiv:hep-ph/0609243].
 - [4] Y. Koide, Phys. Rev. D **71** (2005) 016010 [arXiv:hep-ph/0406286].
 - [5] C. Buttar *et al.*, arXiv:hep-ph/0604120.
 - [6] N. Marinelli, arXiv:0709.2335 [hep-ex].
 - [7] K. S. Babu and J. Kubo, Phys. Rev. D **71** (2005) 056006 [arXiv:hep-ph/0411226].
 - [8] P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. A **10** (1995) 4689 [arXiv:hep-ph/9409330].
 - [9] P. H. Frampton and T. W. Kephart, Phys. Rev. D **64** (2001) 086007 [arXiv:hep-th/0011186].
 - [10] G. Seidl, arXiv:hep-ph/0301044;
 - [11] K. R. S. Balaji, M. Lindner and G. Seidl, Phys. Rev. Lett. **91** (2003) 161803 [arXiv:hep-ph/0303245].
 - [12] W. Grimus and L. Lavoura, Phys. Lett. B **572** (2003) 189 [arXiv:hep-ph/0305046].
 - [13] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP **0407** (2004) 078 [arXiv:hep-ph/0407112].
 - [14] C. Hagedorn and W. Rodejohann, JHEP **0507** (2005) 034 [arXiv:hep-ph/0503143].
 - [15] C. Hagedorn, M. Lindner and F. Plentinger, Phys. Rev. D **74** (2006) 025007 [arXiv:hep-ph/0604265].
 - [16] Y. Kajiyama, J. Kubo and H. Okada, Phys. Rev. D **75** (2007) 033001 [arXiv:hep-ph/0610072].
 - [17] S. L. Chen and E. Ma, Phys. Lett. B **620** (2005) 151 [arXiv:hep-ph/0505064].
 - [18] M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto, Phys. Rev. D **71** (2005) 011901 [arXiv:hep-ph/0409187].
 - [19] S. Okubo, Phys. Rev. D **12** (1975) 3835.

- [20] A. Blum, C. Hagedorn and M. Lindner, arXiv:0709.3450 [hep-ph].
- [21] J. Kubo, Phys. Lett. B **622** (2005) 303 [arXiv:hep-ph/0506043].
- [22] E. Itou, Y. Kajiyama and J. Kubo, Nucl. Phys. B **743** (2006) 74 [arXiv:hep-ph/0511268].
- [23] J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109** (2003) 795 [Erratum-ibid. **114** (2005) 287] [arXiv:hep-ph/0302196].
- [24] J. Kubo, Phys. Lett. B **578** (2004) 156 [Erratum-ibid. B **619** (2005) 387] [arXiv:hep-ph/0309167].
- [25] A. Mondragon, M. Mondragon and E. Peinado, Phys. Rev. D **76** (2007) 076003 [arXiv:0706.0354 [hep-ph]].
- [26] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477** (1996) 321 [arXiv:hep-ph/9604387].
- [27] M. Ciuchini, E. Franco, A. Masiero and L. Silvestrini, Phys. Rev. D **67** (2003) 075016 [Erratum-ibid. D **68** (2003) 079901] [arXiv:hep-ph/0212397].
- [28] L. Silvestrini, Int. J. Mod. Phys. A **21** (2006) 1738 [arXiv:hep-ph/0510077].
- [29] M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. **97** (2006) 021803 [arXiv:hep-ph/0603114].
- [30] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, arXiv:hep-ph/0703204.
- [31] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, arXiv:0705.3650 [hep-ph].
- [32] A. Lenz, Phys. Rev. D **76** (2007) 065006 [arXiv:0707.1535 [hep-ph]].
- [33] Y. Yamada, Phys. Rev. D **50** (1994) 3537 [arXiv:hep-ph/9401241].
- [34] J. Hisano and M. A. Shifman, Phys. Rev. D **56** (1997) 5475 [arXiv:hep-ph/9705417].
- [35] I. Jack and D. R. T. Jones, Phys. Lett. B **415** (1997) 383 [arXiv:hep-ph/9709364].
- [36] L. V. Avdeev, D. I. Kazakov and I. N. Kondrashuk, Nucl. Phys. B **510** (1998) 289 [arXiv:hep-ph/9709397].
- [37] I. Jack, D. R. T. Jones and A. Pickering, Phys. Lett. B **426** (1998) 73 [arXiv:hep-ph/9712542].
- [38] T. Kobayashi, J. Kubo and G. Zoupanos, Phys. Lett. B **427** (1998) 291 [arXiv:hep-ph/9802267].
- [39] I. Jack, D. R. T. Jones and A. Pickering, Phys. Lett. B **432** (1998) 114 [arXiv:hep-ph/9803405].
- [40] D. I. Kazakov and V. N. Velizhanin, Phys. Lett. B **485** (2000) 393 [arXiv:hep-ph/0005185].
- [41] E. Kraus and D. Stockinger, Phys. Rev. D **65** (2002) 105014 [arXiv:hep-ph/0201247].
- [42] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **229** (1983) 381; Phys. Lett. B **166** (1986) 329.
- [43] N. Maekawa, Phys. Lett. B **282** (1992) 387.
- [44] J. R. Espinosa, J. M. Moreno and M. Quiros, Phys. Lett. B **319** (1993) 505 [arXiv:hep-ph/9308315].
- [45] K. S. Babu and S. M. Barr, Phys. Rev. D **49** (1994) 2156 [arXiv:hep-ph/9308217].
- [46] M. Masip and A. Rasin, Phys. Rev. D **52** (1995) 3768 [arXiv:hep-ph/9506471]; Nucl. Phys. B **460** (1996) 449 [arXiv:hep-ph/9508365].

- [47] N. Haba, M. Matsuda and M. Tanimoto, Phys. Rev. D **54** (1996) 6928 [arXiv:hep-ph/9512421].
- [48] S. W. Ham, S. K. Oh and H. S. Song, Phys. Rev. D **61** (2000) 055010 [arXiv:hep-ph/9910461].
bibitemBranco:2000dq G. C. Branco, F. Kruger, J. C. Romao and A. M. Teixeira, JHEP **0107** (2001) 027 [arXiv:hep-ph/0012318].
- [49] C. Hugonie, J. C. Romao and A. M. Teixeira, JHEP **0306** (2003) 020 [arXiv:hep-ph/0304116].
- [50] M. Maniatis, A. von Manteuffel and O. Nachtmann, arXiv:0707.3344 [hep-ph].
- [51] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
- [52] M. Bona *et al.* [UTfit Collaboration], JHEP **0610** (2006) 081.
- [53] H. D. Kim, S. Raby and L. Schradin, Phys. Rev. D **69** (2004) 092002 [arXiv:hep-ph/0401169].
- [54] O. Felix, A. Mondragon, M. Mondragon and E. Peinado, AIP Conf. Proc. **917** (2007) 383 [Rev. Mex. Fis. **S52N4** (2006) 67] [arXiv:hep-ph/0610061].
- [55] Y. Kajiyama, JHEP **0704** (2007) 007 [arXiv:hep-ph/0702056].
- [56] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B **267** (1986) 415.
- [57] F. Gabbiani and A. Masiero, Nucl. Phys. B **322** (1989) 235.
- [58] S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B **606** (2001) 151 [arXiv:hep-ph/0103320].
- [59] M. Endo, M. Kakizaki and M. Yamaguchi, Phys. Lett. B **583** (2004) 186 [arXiv:hep-ph/0311072].
- [60] J. Hisano, Nucl. Phys. Proc. Suppl. **137** (2004) 169 [arXiv:hep-ph/0405185].
- [61] J. Hisano and Y. Shimizu, Phys. Rev. D **70** (2004) 093001 [arXiv:hep-ph/0406091].
- [62] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, arXiv:0711.4022 [hep-ph].
- [63] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].
- [64] M. Ciuchini, E. Franco, V. Lubicz, G. Martinelli, I. Scimemi and L. Silvestrini, Nucl. Phys. B **523** (1998) 501 [arXiv:hep-ph/9711402].
- [65] M. Artuso *et al.* [CLEO Collaboration], Phys. Rev. Lett. **95** (2005) 251801 [arXiv:hep-ex/0508057].
- [66] D. Asner *et al.* [Heavy Flavor Averaging Group], <http://www.slac.stanford.edu/xorg/hfag/charm/index.html>
- [67] E. Dalgic *et al.*, Phys. Rev. D **76** (2007) 011501 [arXiv:hep-lat/0610104].
- [68] N. Tantalo, arXiv:hep-ph/0703241.
- [69] M. Della Morte, PoS **LAT2007** (2007) 008 [arXiv:0711.3160 [hep-lat]].
- [70] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204** (2002) 025 [arXiv:hep-lat/0110091].
- [71] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97** (2006) 211802 [arXiv:hep-ex/0607021].
- [72] A. Lenz and U. Nierste, JHEP **0706** (2007) 072 [arXiv:hep-ph/0612167].
- [73] Y. Aoki *et al.*, Phys. Rev. D **73** (2006) 094507 [arXiv:hep-lat/0508011].
- [74] D. Becirevic and G. Villadoro, Phys. Rev. D **70** (2004) 094036 [arXiv:hep-lat/0408029].
- [75] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173

- [arXiv:hep-ph/0307344].
- [76] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631
[arXiv:hep-ph/9808385].